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VOL. 18, SEC. A. MAY, 1940 NUMBER 5

THE SPECIFIC HEAT OF MANGANESE FROM 16° TO 22° K[1]

BY R. G. ELSON[2], H. GRAYSON SMITH[3] AND J. O. WILHELM[4]

Abstract

A calorimeter is described for routine measurements of specific heats in the temperature region of liquid hydrogen and liquid helium. It is designed so that samples can be interchanged without disturbing the calibration of the thermometer, or the water equivalent of the calorimeter. The calorimeter has been used to measure the specific heat of manganese from 16° to 22° K. It was found that the atomic heat of this metal is given by the formula

$$C = 0.00421T + 464\left[\frac{T}{410}\right]^3.$$

Introduction

Although many designs of apparatus for the measurement of specific heats at very low temperatures have been developed, on the whole, they have been complicated and usually adaptable to only one type of specimen. The apparatus to be described in this article is believed by the authors to incorporate simplicity and speed with reasonable accuracy, and is adapted to solid and powdered specimens.

Changes of temperature are measured by means of a constantan resistance thermometer, using a low resistance Mueller bridge with compensated leads. Since the method involves the measurement of the changes in temperature in short intervals of time, a tape chronograph measuring to one one-hundredth of a second is employed for the time scale.

In this paper the apparatus is described, with an account of measurements made with manganese metal down to 16° K.

Description of Apparatus

Fig. 1 shows the arrangement of the calorimeter system with its vacuum jacket, which is immersed in liquid hydrogen or liquid helium. The three leads, *B*, from the Mueller bridge and the two leads, *H*, for the heating coil enter the middle branch of the vacuum system through an insulating vacuum seal. Twisted and shellacked together, they continue down the centre of the tube, and are cemented into a small copper tube, *U*, which is soldered to the wall as shown. On leaving *U* the leads are separated, and wound and

[1] *Manuscript received February 1, 1940.*
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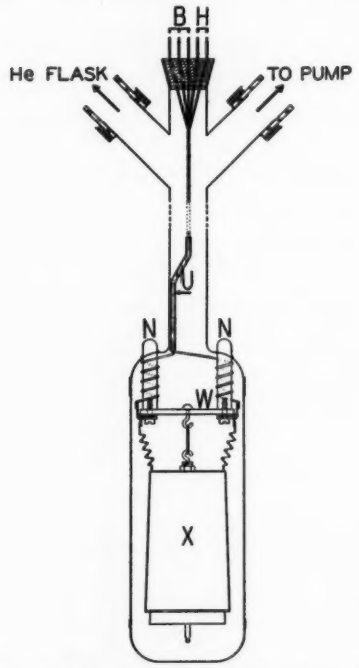

FIG. 1. *Calorimeter system with its vacuum jacket.*

cemented around three solid copper rods, N (only two shown in figure), thence to a five point distribution block, where the five leads from the calorimeter system, X, (only two shown) meet the incoming leads. The purpose of the tube, U, and the copper rods, N, is to reduce conduction of heat down the lead-in wires, by bringing them to the temperature of the cryostat bath. The copper plate, W, prevents heat radiated down the central tube from striking the calorimeter system. The calorimeter system, X, is supported by a fine linen thread from a single hook.

Fig. 2 shows a cross-section of the calorimeter system. It consists of three accurately fitted copper cones. The inner cone, K, encloses the specimen, S, to be tested. The cone, O, is the heater-thermometer unit, while the cone, R, is a very thin, highly polished radiation shield. The cone, O, is slotted to receive the thermometer and heater wires, which are non-inductively wound, and cemented alternately around it along its whole length.

If a sufficiently large solid specimen is obtainable it may be machined and used to replace the cone, K. If, however, the specimen is in a powdered form, as in the present case, then it is compressed into K by the compression plate and nut, P. To improve the thermal conduction throughout the compressed powdered specimen the top of K is soldered into place. K is evacuated

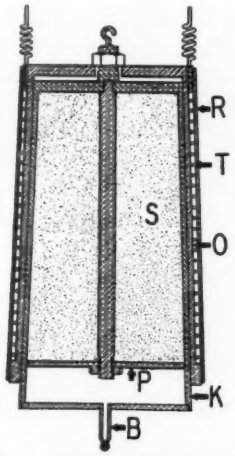

FIG. 2. *Cross-section of the calorimeter system.*

through *B*, flushed with helium gas, and finally *B* is pinched off and soldered, leaving an atmosphere of helium gas in the container.

The heater and resistance thermometer are made from No. 40, D.S.C. constantan (Eureka) wire, and are cemented around *O* with "Glyptol" cement. Changes in the resistance of the thermometer can be detected to one ten-

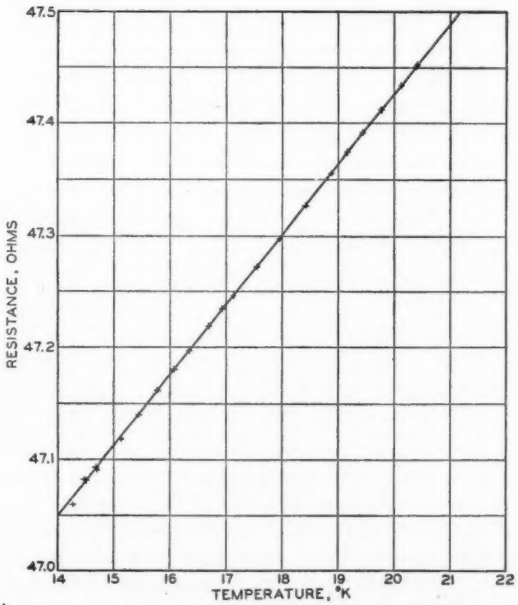

FIG. 3. *Calibration of the constantan resistance thermometer.*

thousandth of an ohm using the Mueller bridge and a Kipp and Zonen galvanometer with telescope and scale (sensitivity = 1000 cm. per ohm). The resistance of the thermometer at room temperature (21° C.) is 49.7132 ohms. Between 14° and 22° K the change in resistance of the thermometer with change of temperature was found to be very nearly linear, having a slope of 15.92 degrees per ohm (Fig. 3). The thermometer was calibrated in the region 14° to 22° K by comparison with a hydrogen vapour pressure thermometer based on the relation developed by Keesom, Bijl, and van den Horst (1), viz.,

$$t = -260.865 + 1.0619 \log_{10} p + 1.7233 \log_{10}^2 p,$$

where p is in centimetres of mercury and t is in degrees Centigrade. The calibration of the resistance thermometer was checked over a three month period and no noticeable change was found.

Experimental Method

Before immersion of the apparatus in the low temperature bath the system, Fig. 1, was evacuated, flushed with helium gas and left with an atmosphere of helium surrounding X. The apparatus was then immersed in the bath, in this case liquid hydrogen, so that the bath level was well above the protruding ends of the rods, N. When the thermometer reached its equilibrium temperature the apparatus was again evacuated, and kept thus during the experiment.

Beginning at the lowest obtainable temperature, simultaneous readings of the resistance and the time were made before, during, and after heating the specimen. Before the calorimeter system returned to its original low temperature, the temperature of the bath was raised, and the experiment was repeated at a slightly higher mean temperature. Thus the temperature of the surroundings was never far below that of the calorimeter.

The true rise of temperature during the heating period was found by the graphical method developed by Keesom and Kok (2). The temperature was plotted against the time, before, during, and after each heating period. Tangents were drawn to the pre-heating curve and to the cooling curve at their points of intersection with the heating curve. If the heating curve is a straight line, as was usually the case, the corrected increase of temperature is the ordinate enclosed between the two tangents at the mid-point of the heating period. Owing to a slight lag in attaining thermal equilibrium throughout the sample, it was difficult to draw the tangent to the cooling curve by inspection. The cooling curve was found to obey Newton's law of cooling accurately once thermal equilibrium was established, and consequently this law was used to extrapolate the cooling curve back to intersect the heating curve, in order to find the slope of the tangent more accurately.

In the determination of the water equivalents of the empty calorimeter system between 15° and 22° K, the calorimeter was sealed with a definite amount of solder and a definite length of seal-off tube. These conditions were reproduced exactly when the calorimeter was filled with manganese.

TABLE I

Weight of calorimeter system (empty), 67.7564 gm.
Weight of powdered manganese, 50.8551 gm.

Mean temp., °K	Time of heating, sec.	Heat input, cal.	Temp. rise, °K	Water equiv. of calorimeter, cal.-deg.$^{-1}$	Heat absorbed by manganese, cal.	Specific heat of manganese, cal.-mole^{-1}-deg.$^{-1}$
15.90	8.65	1.296×10^{-1}	0.8185	7.050×10^{-2}	7.19×10^{-2}	9.48×10^{-2}
15.91	10.90	1.306	0.8285	7.064	7.21	9.39
16.03	13.53	1.203	0.7352	7.220	6.72	9.88
16.67	14.57	1.300	0.7280	8.088	7.11	10.55
16.85	14.22	1.269	0.7092	8.335	6.78	10.31
17.16	15.34	1.366	0.7335	8.805	7.20	10.60
17.32	19.44	1.728	0.8995	9.064	9.13	10.96
17.25	14.65	1.302	0.7003	8.960	6.75	10.37
17.41	15.65	1.391	0.7209	9.220	7.26	10.88
17.57	15.96	1.419	0.7216	9.482	7.35	10.99
17.73	16.29	1.448	0.7164	9.760	7.49	11.29
17.80	13.52	1.202	0.5732	9.885	6.35	11.98
17.79	13.03	1.158	0.5703	9.880	5.95	11.26
18.27	14.76	1.593	0.7331	10.73	8.06	11.88
18.51	15.18	1.635	0.7430	11.16	8.06	11.71
18.75	15.63	1.684	0.7340	11.61	8.32	12.24
18.99	16.40	1.772	0.7423	12.06	8.77	12.77
19.23	14.59	1.760	0.7257	12.53	8.51	12.66
19.31	14.74	1.777	0.7283	12.69	8.53	12.65
19.35	10.81	1.304	0.5289	12.77	6.29	12.83
19.87	14.10	1.918	0.7328	13.87	9.02	13.28
19.96	13.20	1.796	0.6689	14.08	8.54	13.79
19.97	13.05	1.772	0.6735	14.10	8.23	13.19
20.14	13.78	1.873	0.6923	14.47	8.71	13.59
20.28	12.98	1.934	0.6971	14.78	9.04	14.01
20.41	13.76	2.049	0.7236	15.10	9.57	14.22
20.59	13.70	2.040	0.6949	15.52	9.62	14.95
20.74	14.52	2.163	0.7247	15.89	10.11	15.07
20.91	14.16	2.109	0.6907	16.32	9.82	15.32
20.97	11.38	1.694	0.5580	16.49	7.74	14.98
21.50	14.72	2.181	0.6572	17.93	10.02	16.46
21.85	16.19	2.406	0.7268	18.95	10.29	15.29
22.10	17.32	2.581	0.7370	19.66	11.31	16.58

Table I gives the values obtained for the specific heat of powdered manganese from 16° to 22° K. In this table are listed the times of heating, the heat input, and the corrected rise in temperature during heating. The values of the water equivalent of the calorimeter system were taken from the experimental graph obtained for the empty calorimeter. The sixth column gives the heat absorbed by the manganese and is determined by subtracting the product of Columns 4 and 5 from Column 3. The last column gives the final values of the specific heat in calories per mole per degree.

Discussion of Results

Fig. 4 shows the relation between the specific heat, C, and the mean temperature, T, during the heating period. At low temperatures the specific heat of a metal should, theoretically, approach the form,

$$C = aT + bT^3. \qquad (1)$$

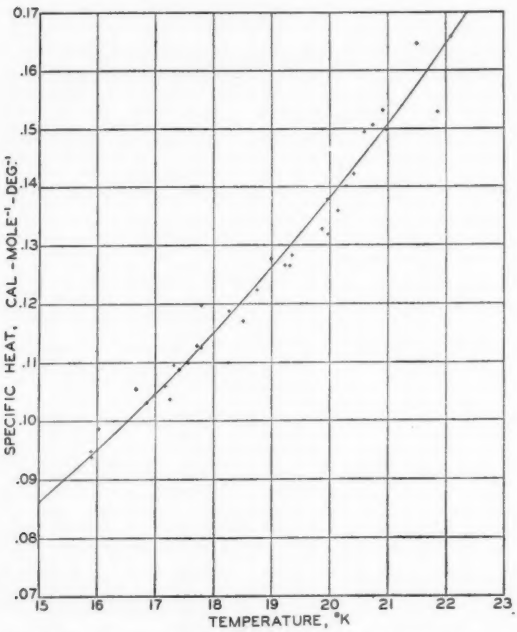

FIG. 4. *Relation between specific heat and mean temperature for manganese.*

In order to test the experimental measurements, the values of $\frac{C}{T}$ were plotted against T^2. This gave very nearly a straight line, from which the best values of the constants, a and b, were readily found, giving the final relation for the specific heat,

$$C = 0.00421\ T + 464\left[\frac{T}{410}\right]^3. \qquad (2)$$

The curve in Fig. 4 is plotted from Equation (2), and the crosses give the individual measurements. The average deviation of a single experimental observation from the calculated values was ± 0.0024 cal. per mole per degree, and the probable error of any single observation was ± 0.0021. Less than 40% of this error is accounted for as instrumental, the greater part being due to the personal error involved in judging the directions of the tangents when calculating the true rise in temperature.

In the theoretical Equation (1) the linear term, aT, represents the specific heat of the conduction electrons. The experimental value, $0.00421T$, is one of the largest electronic specific heats so far observed. However, it is probably reasonable for manganese, in which the unfilled d-shell of electrons may be expected to make a large contribution to the specific heat. The second term represents the specific heat of the crystal lattice, giving the Debye characteristic temperature, $\Theta = 410°$.

Acknowledgments

The authors wish to express their appreciation of the continued encouragement and advice of Prof. E. F. Burton, and the advice given by Prof. J. Satterly. The senior author wishes to thank Professor Burton for extending to him the facilities of the low temperature laboratory.

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2. KEESOM, W. H. and KOK, J. A. Proc. Acad. Sci. Amsterdam, 35 : 294. 1932.

THE PERMEABILITY OF BUILDING PAPERS TO WATER VAPOUR[1]

By J. D. Babbitt[2]

Abstract

Some measurements on the permeability of various types of building papers to water vapour are reported, with a short discussion of the efficiency of the various papers as vapour barriers.

The use of a material impermeable to water vapour as a vapour barrier on the interior of exterior walls has recently become a requirement of the National Housing Act, Minimum Standards, and an established practice in sound construction throughout Canada. It is essential to provide such a barrier to prevent condensation, especially in the walls of houses equipped with air conditioning or humidifying units. This trend in building has led to a demand for a method by means of which the value of a material as a vapour barrier may be readily estimated. A proposed test method was advocated in an earlier paper (1). A list of the diffusion coefficients of a few materials was included in that paper, and in the present paper this list is extended to embrace examples of all types of building papers that might be encountered. The list is not intended, however, to include all individual papers manufactured in Canada, but rather it is presented with the idea of giving the architect, builder, and consumer a means of estimating the protection to be expected from the different types of paper. It is felt that the onus of establishing the coefficient for any individual paper should lie with the manufacturer, as it does in the case of the thermal conductivity of insulating materials.

As has already been mentioned, the method of test was outlined in a previous publication (1) and will not be repeated here. It will be sufficient to define what is meant by a diffusion coefficient. The diffusion coefficient (or diffusivity) of water vapour through a material is the amount of water vapour passing through unit area of the material in unit time when unit vapour pressure difference is maintained per unit thickness. In the case of a material such as a paper where the thickness is integral and can be altered only by using more than one layer, the diffusion is generally expressed as a diffusance rather than as a diffusivity. The diffusance is the amount of water vapour passing through unit area of the material when unit vapour pressure difference is maintained on the two sides of the material. In this paper the diffusance is used throughout and it is expressed as the number of grams of water vapour diffusing through a square metre of the material in 24 hr. when a vapour pressure difference of one millimetre of mercury is maintained on the two sides of the sample. These units have been chosen after due consideration

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as giving a number of a suitable size and also because the units are convenient for the actual measurement. Since the coefficients are mainly used for comparison either between different materials or between a material and a specified standard, the choice of a metric unit should not lead to any great difficulty. This unit is closely related to that accepted by the Pulp and Paper Industry for sheet materials (2).

All the papers were tested at a temperature in the vicinity of 90° F. The relative humidity within the cell was maintained at zero percent by means of anhydrous calcium chloride, while the exterior atmosphere was kept at 75% relative humidity by means of a saturated sodium chloride solution.

The diffusance of the various papers is given in Table I. The different papers have been roughly grouped into 13 different classes. The distinguishing features of the papers are listed in the column headed "Description". The samples of papers were obtained from several different sources and these have been distinguished by the use of letters. In the cases where two separate samples were measured from one piece of paper the figures I and II have been added. The thickness of the papers is given in the third column. These must not be taken as strictly accurate, as no special precautions were taken in determining the thickness, and they are given simply as an indication of the type of paper. The diffusance as measured is shown in the fourth column.

TABLE I

WATER VAPOUR DIFFUSANCES

No.		Description	Thickness, in.	Diffusance, gm. per 24 hr. per sq. metre per mm. of Hg.
Sheathing papers				
1	A	No. 2 Standard dry sheathing paper—15 lb./400 sq. ft.	0.014	113
2	A	Heavy dry sheathing paper—38 lb./400 sq. ft.	0.035	89
3	A	17 oz. Carpet felt	0.056	110
4	A	14 lb. Plain asbestos paper	0.028	104
Asphalt saturated rag felts				
5	A	10 lb.	0.025	4.20
6	C	10 lb.	0.021	1.54
7	B	10 lb.; weight, 44 lb./432 sq. ft.	0.025	0.65
8	AI	15 lb.; weight, 65 lb./432 sq. ft.	0.034	1.93
9	AII	15 lb.	0.037	2.00
10	B	15 lb.; weight, 65 lb./432 sq. ft.	0.038	1.03
11	C	15 lb.	0.033	0.71
12	BI	20 lb.; weight, 43 lb./216 sq. ft.	0.039	0.30
13	BII	20 lb.; weight, 43 lb./216 sq. ft.	0.035	0.33
14	BI	25 lb.; weight, 54 lb./216 sq. ft.	0.061	16.8
15	BII	25 lb.; weight, 54 lb./216 sq. ft.	0.061	20.4
16	A	30 lb.	0.050	4.17
17	B	30 lb.; weight, 65 lb./216 sq. ft.	0.068	2.50
18	C	30 lb.	0.060	0.34

TABLE I—*Continued*

WATER VAPOUR DIFFUSANCES—*Continued*

No.		Description	Thickness, in.	Diffusance, gm. per 24 hr. per sq. metre per mm. of Hg.
Saturated asbestos felts				
19	A	15 lb.; weight, 60 lb./432 sq. ft.	0.028	8.62
20	AI	20 lb.; weight, 60 lb./324 sq. ft. Roofing felt—one surface coated and dusted with sand	0.032	0.11
21	AII	20 lb. Asbestos built-up roofing felt—saturated, one surface coated and sanded	0.034	0.09
22	A	50 lb. Asbestos waterproofing felt—saturated—not coated	0.051	0.25
23	A	15 lb. Tar saturated asbestos felt	0.029	1.11
Asphalt saturated and coated sheathing felts				
24	A	"Regal"—Coated and dusted on one surface 39 lb./200 sq. ft.	0.038	0.14
25	A	"Super-regal"—Heavy weight sheathing coated and dusted on one surface 50 lb./200 sq. ft.	0.031	0.19
26	C	"Supertite Sheathing—Light". Both surfaces coated and dusted	0.036	0.09
27	C	"Supertite Sheathing—Heavy". Both surfaces coated and dusted	0.045	0.06
28	B	"Light" Both surfaces coated and dusted—30 lb./200 sq. ft.	0.036	0.11
29	B	"Super" Both surfaces coated and dusted—36 lb./200 sq. ft.	0.045	0.04
30	A	Base felt. 45 lb./108 sq. ft. Coated both surfaces, dusted one surface	0.074	0.05
31	A	High grade saturated and coated sheathing: rag felt base saturated and coated on one surface. Talc dusted on coated surface.	0.036	0.10
Asphalt saturated sheathing papers				
32	C	No. 2	0.017	0.95
33	A	No. 2—25 lb./400 sq. ft.	0.015	10.6
34	A	Asphalt saturated rag felt sheathing paper	0.017	1.83
35	B	"Renown" asphalt sheathing—25 lb./400 sq. ft.	0.017	2.75
36	B	Asphalt kraft sheathing—25 lb./400 sq. ft.	0.015	1.02
Asphalt saturated and coated kraft papers (glossy surface)				
37	B	Saturated and surface coated—45 lb./500 sq. ft.	0.013	0.12
38	C	"Black Insulo Waterproof Paper"—Saturated and coated	0.019	0.27
39	A	Saturated and surface coated—9 lb./100 sq. ft.	0.018	0.32

TABLE I—*Continued*

WATER VAPOUR DIFFUSANCES—*Continued*

No.		Description	Thickness, in.	Diffusance, gm. per 24 hr. per sq. metre per mm. of Hg.
Asphalt coated kraft papers (not saturated)—glossy surface				
40	BI	Coated building paper—Heavy—24 lb./500 sq. ft.	0.013	0.27
41	BII	Coated building paper—Heavy—24 lb./500 sq. ft.	0.012	0.49
*42	A	Asphalt coated kraft building paper—45 lb./500 sq. ft.	0.008	0.96
Tar-saturated rag felts				
43	C	7 oz. Tarred felt	0.029	18.5
44	B	7 oz. Tarred felt—50 lb./400 sq. ft.	0.032	4.77
45	A	7 oz. Standard tar-saturated felt—50 lb./400 sq. ft.	0.031	0.44
46	A	7 oz. Tar-saturated rag felt	0.028	10.0
47	C	10 oz. Tarred felt	0.041	10.6
48	B	10 oz. Tarred felt—45 lb./300 sq. ft.	0.030	18.3
49	C	16 oz. Tarred felt	0.056	13.3
50	B	16 oz. Tarred felt—50 lb./200 sq. ft.	0.047	0.79
51	A	16 oz. Tar-saturated rag felt	0.060	22.4
52	C	15 lb. Tarred felt	0.042	25.5
53	B	15 lb. Tarred felt—65 lb./432 sq. ft.	0.032	3.77
54	A	15 lb. Tar-saturated rag felt	0.033	9.66
Tar-saturated sheathing papers				
55	C	No. 2 Tarred sheathing	0.018	15.0
56	A	No. 2 Tar-saturated sheathing paper 25 lb./400 sq. ft.	0.016	6.55
57	A	Tar-saturated sheathing	0.016	8.69
58	AI	Heavy tar-saturated sheathing paper 45 lb./400 sq. ft.	0.040	12.3
59	AII	Heavy tarred sheathing	0.032	8.11
60	B	"Renown" tarred sheathing—25 lb./400 sq. ft.	0.022	7.87
61	B	Heavy tarred sheathing—50 lb./400 sq. ft.	0.031	16.5
Duplex papers (asphalt between two sheets of kraft paper)				
62	AI	Waterproof sheathing—16 lb./500 sq. ft.	0.009	0.31
63	AII	Waterproof sheathing—16 lb./500 sq. ft.	0.008	0.38
64	A	Reinforced waterproof sheathing—reinforced with extra heavy jute cord, criss-crossed to give extra strength	0.008	1.04
65	B	Fibreen—reinforced duplex building paper	0.016	0.40
66	B	Treated Fibreen—unaffected by acids in ensilage	0.018	0.23
67	E	30-30-30 Paper	0.007	0.53
68	F	30-30-30 Paper	0.008	0.51

* *This sample was old and in places paper was visible through the asphalt.*

TABLE I—*Concluded*

WATER VAPOUR DIFFUSANCES—*Concluded*

No.		Description	Thickness, in.	Diffusance, gm. per 24 hr. per sq. metre per mm. of Hg.
Waxed papers				
69	C	Light waxed kraft sheathing	0.004	0.79
70	C	Medium waxed kraft sheathing	0.006	0.24
71	C	Heavy waxed kraft sheathing	0.007	3.22
72	B	Light waxed kraft sheathing—9 lb./450 sq. ft.	0.003	0.07
73	B	Medium waxed kraft sheathing—12.5 lb./450 sq. ft.	0.005	0.10
74	B	Heavy waxed kraft sheathing—18 lb./450 sq. ft.	0.007	0.36
75	D	Light weight waxed paper	0.004	0.74
76	D	Medium weight waxed paper	0.006	0.27
77	D	Heavy weight waxed paper	0.008	0.88
78	A	Light grade waxed sheathing paper—2 lb./100 sq. ft.	0.004	0.08
79	AI	Medium grade waxed sheathing paper—2.8 lb./100 sq. ft.	0.005	1.80
80	AII	Medium grade waxed sheathing paper—2.8 lb./100 sq. ft.	0.005	2.01
81	A	Heavy grade waxed sheathing paper—4 lb./100 sq. ft.	0.007	2.16
Heavy roofing papers				
82	B	Cap Sheet, Medium—42 lb./108 sq. ft.	0.076	0.05
83	B	45 lb. Base Sheet—42 lb./108 sq. ft.	0.080	0.23
84	B	Janus Roofing, Light—35 lb./108 sq. ft.	0.052	0.06
85	B	Janus Roofing, Medium—45 lb./108 sq. ft.	0.066	0.05
86	B	Janus Roofing, Heavy—55 lb./108 sq. ft.	0.073	0.04
87	B	Janus Roofing, Extra Heavy—65 lb./108 sq. ft.	0.105	0.06
88	B	Janus Roofing, Super Heavy—75 lb./108 sq. ft.	0.115	0.02
89	B	Talisman Roofing, Medium—45 lb./108 sq. ft.	0.073	0.04
90	B	Talisman Roofing, Heavy—55 lb./108 sq. ft.	0.088	0.04
91	B	Talisman Roofing, Extra Heavy—65 lb./108 sq. ft.	0.109	0.09
92	B	Talisman Roofing, Super Heavy—75 lb./108 sq. ft.	0.126	0.07
Infused papers				
93	G	Waxed kraft paper, asphalt infusion on one surface, surface ribbed for application of rock wool	0.006	0.34
94	A	Kraft paper, unwaxed, asphalt infused one surface (tested after rock wool attached)	0.005	0.54
95	H	Kraft paper, waxed, coated one surface and ribbed	0.007	0.34
96	H	Kraft paper, waxed, coated one surface	0.009	0.29
97	I	Kraft paper, unwaxed, infused one surface (tested after rock wool attached)	0.005	0.52

Discussion

It is interesting to discuss in detail the results obtained with the different classes of papers.

1. *Sheathing papers:*—These are dry papers and their resistance to vapour movement is almost negligible. The results for these papers must be taken only as indicating the order of magnitude to be expected, as the diffusance is

so large that it is impossible to obtain accurate results without additional precautions.

2. *Asphalt saturated rag felts:*—There is a wide variation in the diffusance through the different papers in this class. It is obvious that an asphalt felt cannot be relied on as a vapour barrier. There does not seem to be any correlation between thickness or weight and diffusance, but this may be masked by large variations in the individual samples.

3. *Asphalt saturated asbestos felts:*—Wide variations are obtained for the diffusances of the asbestos felts. Three samples have low values and it should be noticed that two of these are asphalt coated and the third is a heavy thick paper.

4. *Asphalt saturated and coated sheathing felts:*—All these felts have low vapour diffusances. The difference between these papers and those in Class 2 is apparently caused by the surface coating.

5. *Asphalt saturated sheathing papers:*—These papers are, like the saturated rag felts, relatively permeable to water vapour.

6. *Asphalt saturated and coated kraft papers:*—These papers have a low vapour diffusance and may be considered as vapour barriers. The outstanding feature of these papers is that they have a black glossy surface, which indicates that the asphalt forms a continuous film on the surface. It is this film that prevents the movement of the moisture.

7. *Asphalt coated kraft papers (not saturated):*—These papers are similar in appearance to those in Class 6, but in this case the paper is not saturated. The fact that the resistance to vapour movement is nearly the same as that of the saturated papers confirms the assumption that it is the surface film that is the impermeable portion of the paper.

8. *Tar saturated rag felts:*—Like the asphalt saturated rag felts, these are very permeable to vapour.

9. *Tar saturated sheathing papers:*—All are very permeable.

10. *Duplex papers:*—The duplex papers form an important class of vapour barriers. Paper 64, reinforced waterproof sheathing, is more permeable than the others and could not be classed as a vapour barrier. In this case the water vapour can diffuse through the jute cord and the effectiveness of the asphalt layer is reduced.

11. *Waxed papers:*—A study of this table indicates that it is the light papers that are the efficient vapour barriers. In this case again it is a surface film that plays the predominant role. In the light weight papers the wax is present as a film on the surface, while in the heavy papers the wax is absorbed by the paper and loses its effectiveness as a vapour stop.

12. *The heavy roofing papers:*—As may be expected, these papers, being heavily saturated and coated with asphalt, are all good vapour barriers. The values of the diffusance quoted for these papers must be considered rather as upper limits than as absolute values since these papers are so imper-

vious that the approach to equilibrium is very slow and the changes in weight of the cells are so small that an exact determination would require more care than can be given in an investigation of this nature.

13. *Infused papers:*—These are kraft papers (either waxed or dry) one surface of which is infused with an asphalt preparation. By infusion a light coat of asphalt is applied to the paper with very little penetration. These papers are used mainly as backing for rock wool batts. Considering their lightness and cheapness, they are strong and efficient vapour barriers.

In general it is apparent from these tests that for a paper to be impervious to water vapour it must possess a continuous film of some impermeable material such as wax or asphalt. The presence of the impermeable material as an absorbed phase within the paper is not sufficient; it must be present as a continuous film on the surface, either between two sheets of paper as in the duplex papers or on the actual surface as in the coated papers. The evidence from this work is that the diffusance is independent of all other factors such as thickness, weight, type of paper, etc. The resistance of a continuous film is so much greater than the resistance of the paper itself that the extent and perfection of the film is the predominant factor. The actual amount of the transmission is probably dependent on the number and extent of small pin holes that are present. Once a continuous film without holes is obtained on the paper, the diffusance should vary inversely as the thickness of this film.

The amount of diffusance that is allowable before a paper may be used as a vapour barrier with the assurance that condensation will be eliminated depends naturally on the outside and inside humidity conditions, the temperature gradient, and the permeability of the other materials used in the wall. It is essential to realize that a paper might, under certain conditions, be sufficiently impermeable to prevent condensation, while, under slightly more severe conditions, the moisture might accumulate in sufficient quantities to cause severe structural damage. However, while in the extreme case each problem should be treated individually, it is possible to indicate the order of diffusance that is allowable in a vapour barrier suitable for general conditions. Such estimates have been given by Tyler Stewart Rogers (3), L. V. Teesdale (5), and J. D. Babbitt (1). All these estimates are approximately the same and when allowance is made for the different units they represent a diffusance of less than 0.5 gm./24 hr./sq. metre/mm. of mercury. This would seem then to be a suitable figure at which to set the minimum requirements for a vapour barrier until such time as experience shall indicate otherwise. It is interesting to note that under these conditions the following types of papers may be classified as vapour barriers.

1. Asphalt saturated and coated sheathing felts,
2. Asphalt saturated and coated kraft papers,
3. Asphalt coated kraft papers,
4. Duplex papers,

5. Light weight waxed kraft papers in which the wax is present as a film on the surface,
6. Heavy roofing papers,
7. Infused papers.

Acknowledgments

The author wishes to express his appreciation of the work of D. K. Stiles, laboratory assistant at the National Research Laboratories, Ottawa, who has been responsible for the preparation and weighing of the samples studied in this paper. He also wishes to acknowledge the suggestions received in discussions with Mr. John J. Hyland of the National Housing Administration.

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NOTE ON A METHOD OF PLOTTING ELECTRON DISTRIBUTION CURVES FOR THE *F* LAYER[1]

By C. W. McLeish[2].

Abstract

A method of analysis of a (P', f) record is described, which enables one to plot the electron density of the F layer of the ionosphere with respect to height above its lower boundary.

Introduction

Magnetic and auroral disturbances have been correlated with ionosphere storms. Such storms are detected, with multifrequency sounding apparatus, as abrupt changes in critical, or penetration, frequency of the F and other layers. The purpose of this paper is to describe a method of analyzing (P', f) records to obtain not only changes in maximum electron density, given by the critical frequency, but also changes in electron distribution. Previous methods (1) of finding electron distribution entailed the measuring of reflection coefficients and equivalent heights for various angles of incidence. One obvious disadvantage is that it is practically impossible to obtain instantaneous observations from various angles of incidence. Recent improvements (2) in the accuracy of equivalent height measurements will make possible the plotting of a dependable distribution curve from a single (P', f) record.

In order to establish the procedure, it may first be desirable to review some of the properties of a dispersive medium in a magnetic field.

Review

Total reflection of radio waves in an ionized medium occurs when the group velocity becomes zero. For zero group velocity, U, the refractive index, μ, must be either zero or infinity (3). The exact expression for μ contains a term that depends on the collisional frequency of the electrons in the ionized medium. This term may be neglected at frequencies of the order of the critical frequency of the F layer. The approximate expression for electron density at the point of reflection is then

$$N = \frac{\pi m f^2}{e^2} = 1.24 \times 10^{-8} f^2 \text{ electrons per cc.}, \qquad (1)$$

where e and m are electron charge and mass respectively, and f is the frequency of the incident waves.

The presence of the earth's magnetic field in the ionosphere produces the inverse Zeeman effect. The incident ray is split into two oppositely polarized

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waves with different group velocities, refractive indices, and absorption coefficients. For a field, H,

$$N = \frac{\pi m}{e^2} f_o^2, \tag{2}$$

and

$$N = \frac{\pi m}{e^2} (f_x^2 \pm f_x f_H), \tag{3}$$

where f_o refers to the ordinary ray, which rotates anticlockwise looking in the direction of propagation of a downcoming ray in the Northern Hemisphere; and f_x refers to the extraordinary, or clockwise, ray. f_H is the electron gyro frequency, and is equal to $\frac{He}{2\pi mc}$. In Equation (3), the positive sign holds when $f_x < f_H$. Equations (1), (2), and (3) are derived from the condition that $\mu = 0$. When $\mu \to \infty$, conditions for total reflection require values of N greater than those for $\mu = 0$, so reflection must always take place according to the above equations.

Differential Absorption Retardation

From Equations (2) and (3), ordinary and extraordinary rays are both reflected from the same level in the layer when

$$f_o^2 = f_x^2 - f_x f_H, \tag{4}$$

f_x being greater than f_H in this case (Fig. 1).

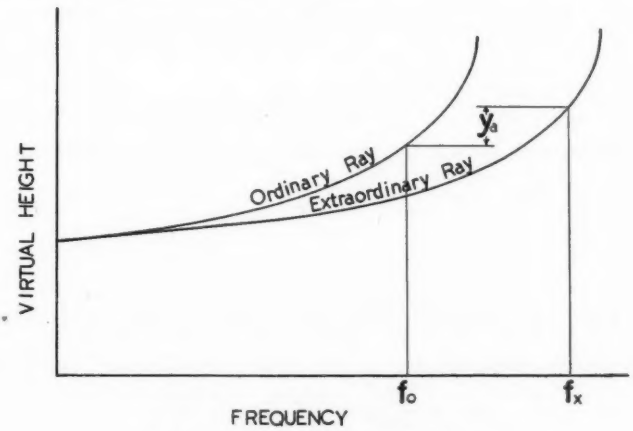

FIG. 1. *Virtual height curves for F layer.*

Assume longitudinal propagation with respect to the earth's field for vertical incidence of the waves on the layer. In dealing with the F layer, it must also be assumed that the E layer has very little effect on the difference of retardations of ordinary and extraordinary echoes from the F layer. This is a legitimate assumption for frequencies just below the F layer critical frequency,

and well above the E layer critical frequency. For longitudinal propagation (4)

$$\mu^2 = 1 - \frac{p_o^2}{p(p \mp p_L)}, \tag{5}$$

where $p_o^2 = \frac{4\pi Ne^2}{m}$, and $p_L = \frac{H_L e}{mc}$. The upper sign refers to the extraordinary ray.

Expanding Equation (5),

$$\left.\begin{aligned} \mu_o &\doteq 1 - \tfrac{1}{2}\frac{p_o^2}{p(p + p_L)} = 1 - \frac{2\pi Ne^2}{mp(p + p_L)}, \\ \text{and} \\ \mu_x &\doteq 1 - \tfrac{1}{2}\frac{p_o^2}{p(p - p_L)} = 1 - \frac{2\pi Ne^2}{mp(p - p_L)}. \end{aligned}\right\} \tag{6}$$

Let the electron distribution in the layer be such that $N = N_o f(y)$, where N_o is the electron density at the height of reflection, y_o, above the lower boundary of the layer. Substituting for N in Equation (6),

$$\left.\begin{aligned} \mu_o &= 1 - \frac{2\pi N_o e^2}{mp(p + p_L)} \cdot f(y), \\ \text{and} \\ \mu_x &= 1 - \frac{2\pi N_o e^2}{mp(p - p_L)} \cdot f(y). \end{aligned}\right\} \tag{7}$$

Now the group velocity, U, of a wave train in the medium may be expressed (4) as $\frac{1}{U} = \frac{d}{dp}\left(\frac{p}{v}\right)$, where v is the phase velocity. Then,

$$\left.\begin{aligned} \frac{1}{U_o} &= \frac{d}{dp}\left(\frac{p\mu_o}{c}\right) = \frac{1}{c} + \frac{2\pi N_o e^2}{mc} f(y) \cdot \frac{1}{(p_o + p_L)^2}, \\ \text{and} \\ \frac{1}{U_x} &= \frac{d}{dp}\left(\frac{p\mu_x}{c}\right) = \frac{1}{c} + \frac{2\pi N_o e^2}{mc} f(y) \cdot \frac{1}{(p_x - p_L)^2}. \end{aligned}\right\} \tag{8}$$

The difference in time for the two components to reach a certain height y_o is $\int_o^{y_o} \frac{dy}{U_x} - \int_o^{y_o} \frac{dy}{U_o}$.

Therefore

$$y_a = c\int_o^{y_o}\left(\frac{1}{U_x} - \frac{1}{U_o}\right)dy = \frac{2\pi N_o e^2}{m}\left[\frac{1}{(p_x - p_L)^2} - \frac{1}{(p_o + p_L)^2}\right]\int_o^{y_o} f(y)dy \tag{9}$$

But N_o is the electron density at the point of reflection, and is given by

$$N_o = \frac{mp_o^2}{4\pi e^2} = \frac{m(p_x^2 - p_x p_L)}{4\pi e^2}. \tag{10}$$

Then from Equation (9) we have

$$y_a = \frac{p_o^2}{2}\left[\frac{1}{(p_x - p_L)^2} - \frac{1}{(p_o + p_L)^2}\right]\int_o^{y_o} f(y) \cdot dy \tag{11}$$

But from Equation (10),

$$p_x = \frac{p_L + \sqrt{p_L^2 + 4p_o^2}}{2} = \frac{p_L}{2} + \sqrt{p_o^2 + \left(\frac{p_L}{2}\right)^2}.$$

Therefore

$$y_a = \tfrac{1}{2}\left[\frac{p_o^2}{\left\{\sqrt{p_o^2 + \left(\frac{p_L}{2}\right)^2} - \frac{p_L}{2}\right\}^2} - \frac{p_o^2}{(p_o + p_L)^2}\right]\int_o^{y_o} f(y) \cdot dy. \qquad (12)$$

Letting $\frac{p_L}{p_o} = X$, and $\frac{y_a}{\int_o^{y_o} f(y) \cdot dy} = Y$, Equation (12) simplifies to

$$Y = \tfrac{1}{2}\left[\left(\frac{1}{\sqrt{1 + \frac{X^2}{4}} - \frac{X}{2}}\right)^2 - \left(\frac{1}{1+X}\right)^2\right] \qquad (13)$$

The critical frequency for the F layer usually lies within the range of 16.5 to 3.3 mc. per sec., which corresponds to a range of 0.1 to 0.5 for X in Ottawa.* Values of Y for this range have been calculated to four significant figures, and an approximate linear relation has been established that gives a maximum error of less than 1.5% over the entire range (Fig. 2).

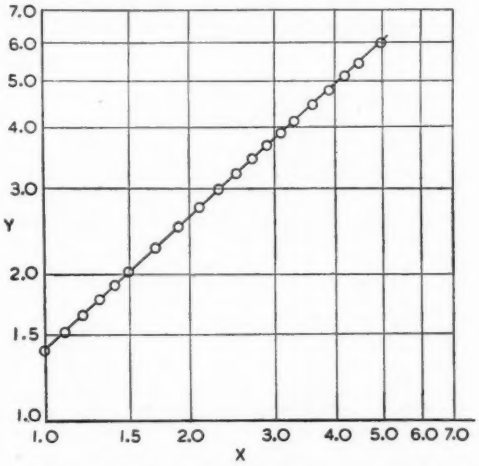

FIG. 2. *Graphical representation of Equation (13).*

The linear relation which shall henceforth be used is

$$Y = 1.145X + 0.0255. \qquad (14)$$

Now we may write

$$y_a = \left(1.145\frac{f_H}{f_o} + 0.0255\right)\int_o^{y_o} f(y) \cdot dy \qquad (15)$$

Chapman shows (5) that a region which exists in an exponential atmosphere, where solar radiation is the ionizing agent, will have a parabolic distribution

* *$H = 0.607$ gauss and angle of dip $= 75.4°$, giving a value of $f_L = 1.65$ mc. per sec.*

of electrons. This is termed a "simple" region. If the F region could be treated as such a region, $f(y)$ would be $\frac{y^2}{y_o^2}$, and

$$y_a = \frac{y_o}{3}\left(1.145\frac{f_H}{f_o} + 0.0255\right). \tag{16}$$

Continuing discussion of the general case, we have, by definition, $f(y) = \frac{N}{N_o}$, so Equation (15) may be rewritten as

$$\frac{y_a}{1.145\frac{f_H}{f_o} + 0.0255} = \frac{1}{N_o}\int_o^{y_o} N \cdot dy,$$

but

$$f_o = \sqrt{\frac{N_o e^2}{\pi m}} \text{ from Equation (2),}$$

so that

$$\frac{y_a}{aN_o^{-\frac{1}{2}} + b} = \frac{1}{N_o}\int_o^{y_o} N \cdot dy, \tag{17}$$

where

$$a = 1.145 f_H \sqrt{\frac{\pi m}{e^2}} = 218,$$

and

$$b = 0.0255.$$

Now $\int_o^{y_o} N \cdot dy$ is the area to the left of the electron distribution curve (Fig. 3).

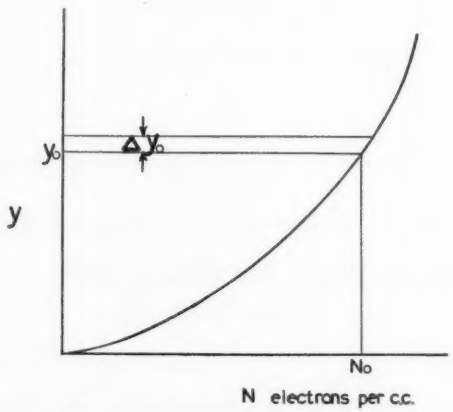

FIG. 3. *Electron distribution curve.*

Therefore

$$\Delta y_o = \frac{1}{N_o}\Delta\left(\int_o^{y_o} N \cdot dy\right) = \frac{1}{N_o}\Delta\left(\frac{N_o y_a}{aN_o^{-\frac{1}{2}} + b}\right) \text{ from Equation (17).}$$

Thus

$$\frac{dy_o}{dN_o} = \frac{1}{N_o} \cdot \frac{d}{dN_o}\left(\frac{N_o y_a}{aN_o^{-\frac{1}{2}} + b}\right).$$

$$\frac{dy_o}{dN_o} = \frac{1}{N_o} \frac{\left(aN_o^{-\frac{1}{2}} + b\right)\left(y_a + N_o \frac{dy_a}{dN_o}\right) - N_o y_a\left(-\frac{1}{2}aN_o^{-\frac{3}{2}}\right)}{(aN_o^{-\frac{1}{2}} + b)^2} \tag{18}$$

Or,

$$\frac{dy_o}{dN_o} = \frac{y_a}{N_o} \cdot \frac{\left(\frac{3}{2}aN_o^{-\frac{1}{2}} + b\right)}{(aN_o^{-\frac{1}{2}} + b)^2} + \frac{dy_a}{dN_o} \cdot \frac{1}{(aN_o^{-\frac{1}{2}} + b)}.$$

The (Y_o, N_o) curve may be arrived at by the "step-by-step" method of drawing tangents of slope given by $\frac{dy_o}{dN_o}$. A procedure suggested by V. A. Bailey (4) appears complicated, and yet is far more rapid for a given accuracy than simply producing tangents. The slope is calculated for $p_o(x_o, y_o)$, and a line drawn to $Q_o(x_o + \frac{\delta}{2}, y_1')$. The slope at Q_o is calculated, and a line drawn from P_o to $P_1(x_o + \delta, y_1)$ with this slope. The operation is repeated, beginning again at P_1. Less than half the operations necessary by the ordinary procedure are needed.

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A NOTE ON THE VELOCITY DISTRIBUTION OF GASEOUS MOLECULES; AND A TABLE FOR OBTAINING VALUES OF THE ERROR FUNCTION COMPLEMENT[1]

BY A. H. HEATLEY[2]

Abstract

The formula for the computation of the fraction of the molecules in a gas that possess energy above a specified amount is simple when motion in only two directions is considered, but becomes difficult to evaluate when three directions are considered. The ratio of the results for the two cases may be easily computed by means of the function $L(x)$, a table of which is given. The relations of $L(x)$ to the error function complement $Erfc(x)$, to the error function $Erf(x)$, and to the Miller and Gordon function $F(x)$ are shown. An extension of the table of $F(x)$ is also given.

Part I. Velocity Distributions

In his discussion of the velocity distribution of the molecules of a gas, Hinshelwood (7, p. 10) compares the results for two distributions, and says "the distribution law so obtained [for two dimensions] cannot give numerical results very different from those yielded by the three-dimensional law, and, as it reduces to a very simple algebraic form, it is sometimes an extremely convenient approximation to use in certain cases instead of the true distribution law". He does not discuss the magnitude of the error involved in the approximation, but gives formulae from which it is readily deduced for the higher velocities.

In the same connection, Farkas and Melville (4, p. 6) say "The evaluation of this complicated formula [for three-dimensional distribution] is rather difficult". The function $L(x)$, which is tabulated in Part II of this paper, permits the easy calculation of the ratio between the results for two and for three dimensions.

Let N be the total number of molecules in a given weight of gas and N_1 the number possessing energy greater than E. Then

$$\frac{N_1}{N} = \frac{4}{\pi^{\frac{1}{2}}} \int_x^\infty t^2 e^{-t^2} dt = \frac{2}{\pi^{\frac{1}{2}}} \left\{ x e^{-x^2} + \int_x^\infty e^{-t^2} dt \right\} \tag{1a}$$

$$= \frac{e^{-x^2}}{x \pi^{\frac{1}{2}}} \left\{ 2x^2 + L(x) \right\}, \tag{1b}$$

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Contribution from the Research Laboratories, Shawinigan Chemicals Ltd., Shawinigan Falls, Que.

[2] *Research Chemist.*

where $x^2 = E/RT$, R is the gas constant, T is the absolute temperature, and $L(x)$ is the function defined and tabulated in Part II.

Equation (1) is derived on the basis of molecular motion in three directions. If motion in only two directions is considered the formula becomes (7, p. 12)

$$\frac{N_1}{N} = e^{-E/RT} = e^{-x^2}. \tag{2}$$

The ratio of the result given by the two equations is designated r:

$$r = \frac{N_1/N \text{ by Eq. (1)}}{N_1/N \text{ by Eq. (2)}} = \frac{2x^2 + L(x)}{x\pi^{\frac{1}{2}}}. \tag{3}$$

Table I gives e^{-x^2}, i.e., N/N_1 by Equation (2), and r for a few values of E/RT. Other values of r may be easily computed from the values of $L(x)$ in Table II. For values of E/RT above 4.4, $r \approx (2x^2 + 1)/x\sqrt{\pi}$, and above 49.0, $r \approx 2x/\sqrt{\pi}$; the error in both cases is less than 1%.

TABLE I

e^{-x^2} and r

E/RT	e^{-x^2}	r	E/RT	e^{-x^2}	r
0	1.000	1.000	16	1.125×10^{-7}	4.651
1	3.679×10^{-1}	1.556	25	1.389×10^{-11}	5.753
4	1.832×10^{-2}	2.512	36	2.320×10^{-16}	6.863
9	1.234×10^{-4}	3.564	49	5.243×10^{-22}	7.978

Part II. The Function $L(x)$

Tables of the error function complement,

$$Erfc(x) = \int_x^\infty e^{-t^2}\,dt, \tag{4}$$

are not suitable for interpolation owing to the rapid decrease to very small values as x increases. In Table II are tabulated values of the function $L(x)$, which tends to unity as a limit as x increases, and is therefore convenient for interpolation.

$$L(x) = 2xe^{x^2}Erfc(x)$$
$$= 1 - \frac{1}{2x^2} + \frac{1 \cdot 3}{(2x^2)^2} - \frac{1 \cdot 3 \cdot 5}{(2x^2)^3} + - \ldots \text{ (Semiconvergent).} \tag{5}$$

The values in the table from $x = 0$ to 3.0 were computed from Kramp's seven-figure table (8)* of $\log e^{x^2}Erf(x)$. From $x = 3.0$ to 5.0, and at $x = 5.5$ and 6.0, they were taken from Burgess' fifteen-place table† (1, p. 321) of $L(x)$. The remaining values were computed by means of Equation (5) to ten places. The nine-place figures were differenced to the fourth or fifth difference to ensure accuracy, and then rounded off to eight-place figures, as tabulated. As a final check the printer's proof was also differenced.

* *The intervals in this table are 0.01. Kramp also tabulates $Erfc(x)$ and $e^{x^2}Erfc(x)$.*

† *The intervals in this table are 0.1. The values given for $H(4.7)$ and $L(4.7)$ are in error. $L(4.7)$ was recomputed by Equation (5).*

TABLE II

$L(x)$

x	$L(x)$	x	$L(x)$	x	$L(x)$
0.0	0.000 00	3.5	0.9633 7793	7.0	0.9900 9340
.1	.158 89	.6	51 9875	.2	06 2140
.2	.286 79	.7	68 9040	.4	11 0862
.3	.390 61	.8	84 6455	.6	15 5911
.4	.475 58	.9	99 3160	.8	19 7646
.5	.545 64	4.0	.9713 0086	8.0	.9923 6382
.6	.603 84	.1	25 8065	.2	27 2397
.7	.652 53	.2	37 7847	.4	30 5939
.8	.693 53	.3	49 0101	.6	33 7227
.9	.728 26	.4	59 5436	.8	36 6459
1.0	.757 87	.5	.9769 4398	9.0	.9939 3809
.1	.783 25	.6	78 7483	.2	41 9435
.2	.805 13	.7	87 5140	.4	44 3478
.3	.824 08	.8	95 7775	.6	46 6065
.4	.840 57	.9	.9803 5760	.8	48 7311
.5	.854 99	5.0	.9810 9431	10.0	.9950 7319
.6	.867 66	.1	17 9096	.5	55 2520
.7	.878 83	.2	24 5037	11.0	59 1797
.8	.888 72	.3	30 7511	.5	62 6137
.9	.897 51	.4	36 6753	12.0	65 6333
2.0	.905 35	.5	.9842 2980	.5	.9968 3024
.1	.912 37	.6	47 6390	13.0	70 6730
.2	.918 67	.7	52 7166	.5	72 7879
.3	.924 34	.8	57 5477	14.0	74 6826
.4	.929 47	.9	62 1478	.5	76 3865
.5	.934 11	6.0	.9866 5311	15.0	.9977 9243
.6	.938 33	.1	70 7110	.5	79 3170
.7	.942 17	.2	74 6997	16.0	80 5821
.8	.945 67	.3	78 5086	.5	81 7348
.9	.948 88	.4	82 1482	17.0	82 7880
3.0	.951 81	.5	.9885 6284	.5	.9983 7528
.1	.954 51	.6	88 9581	18.0	84 6388
.2	.957 00	.7	92 1459	.5	85 4544
.3	.959 29	.8	95 1997	19.0	86 2067
.4	.961 41	.9	98 1267	.5	86 9023
.5	0.963 38	7.0	0.9900 9340	20.0	0.9987 5466

Eight-figure accuracy is far beyond the requirements of Part I of this paper, but may be of value for other applications. Those who require more significant figures or smaller intervals can obtain them from the references quoted above, or below. $L(x)$ can be computed from the error function $Erf(x)$ by means of the relation

$$L(x) = 2xe^{x^2}\frac{\pi^{\frac{1}{2}}}{2}\left\{\frac{2}{\pi^{\frac{1}{2}}}\int_0^{\infty} e^{-t^2}dt - \frac{2}{\pi^{\frac{1}{2}}}\int_0^{x} e^{-t^2}dt\right\} = x\pi^{\frac{1}{2}}e^{x^2}\left\{1 - \frac{2}{\pi^{\frac{1}{2}}}Erf(x)\right\}. \quad (6)$$

The commonly available five-figure tables of $(2/\sqrt{\pi})Erf(x)$ give only three-figure accuracy for $L(x)$ at $x = 2$, but Burgess' fifteen-figure table (1)* gives

* *Unfortunately this table contains some errors (11).*

eleven-figure accuracy at $x = 3$. The intervals are 0.001 and 0.002. A fifteen-figure table of the same function extending to $x = 6$ at intervals of 0.0001 and 0.001 has been computed but not yet published by the "Project for the Computation of Mathematical Tables" (11). Glaisher (5) gives a table of $Erfc(x)$ from $x = 3$ to 4.5, to five or more significant figures at intervals of 0.01. The British Association Committee (3) gives $f(y)$,

$$f(y) = \int_y^\infty e^{\frac{1}{2}(y^2-t^2)}dt = 2^{\frac{1}{2}}e^{y^2/2}\int_{y/2^{\frac{1}{2}}}^\infty e^{-t^2}dt = \frac{1}{y}L(y/\sqrt{2}), \tag{7}$$

to 12 decimal places for $y = 0$ to 10 at intervals of 0.01. Since

$$L(x) = x2^{\frac{1}{2}}f(x2^{\frac{1}{2}}), \tag{8}$$

the highest values of y in these tables correspond to $L(\sqrt{50})$. At $x = 20$, the highest value in the writer's table, only four terms of the series in Equation (5) were required, so that the table can easily be extended as required.

To obtain $Erfc(x)$ from $L(x)$, values of e^x or e^{-x} are required. Such tables are given by Glaisher (6)*, by Becker and van Orstrand (2)**, by Newman (10)†, and by the "Works Projects Administration of New York City"‡ (12).

A convenient partial check on the writer's computations of $L(x)$ from $x = 5.1$ to 12 was afforded by Miller and Gordon's table (9, p. 2878) of the function $F(x)$,

$$F(x) = e^{-x^2}\int_0^x e^{t^2}dt, \tag{9}$$

since the semiconvergent series for $2xF(x)$ consists of the same terms as that for $L(x)$, the signs of the former being all positive, and of the latter, alternately positive and negative. Thus the sum of the two groups of terms (positive and negative) on the right of Equation (5) gives $2xF(x)$ and their difference gives $L(x)$. Because so little additional work was required, a table of $F(x)$ from $x = 10$ to 20 was computed; it is given as Table III.

TABLE III

$F(x)$

x	$F(x)$	x	$F(x)$	x	$F(x)$
10.0	0.0502 53847	13.5	0.0371 39496	17.0	0.0294 62917
10.5	.0478 38014	14.0	.0358 06100	17.5	.0286 18306
11.0	.0456 44752	14.5	.0345 65355	18.0	.0278 20845
11.5	.0436 44541	15.0	.0334 07907	18.5	.0270 66686
12.0	.0418 12876	15.5	.0323 25622	19.0	.0263 52390
12.5	.0401 29249	16.0	.0313 11396	19.5	.0256 74876
13.0	0.0385 76355	16.5	0.0303 58993	20.0	0.0250 31368

* *Nine figures from $x = 0$ to 500.*

** *Seven to nine figures from $x = 0$ to 100.*

† *Twelve to eighteen decimal places. $x = 0$ [.001] 15.35 [.002] 17.30 [.005] 27.64 [.01] 37.00.*

‡ *Twelve to eighteen decimal places. $x = -2.5$ [.0001] 2.500 [.001] 5.00 [0.01] 10.0.*

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MEASUREMENT OF VISCOSITY IN THE CRITICAL REGION. ETHYLENE[1]

By S. G. Mason[2] and O. Maass[3]

Abstract

A precision oscillating disc viscometer has been constructed with a view to studying viscosities of systems at the critical temperature and critical pressure. A new method of computing logarithmic decrements is described and a number of results obtained with ethylene are presented. Of particular interest is the transition in viscosity on an isochore through the critical temperature.

Introduction

Precision measurements of a number of physical properties, such as density (4), adsorption (3), dielectric constant (5), specific heat (8), surface tension (15), in the critical region have been the subject of a number of communications from this laboratory. These measurements have indicated the necessity for modifying the classic theory of continuity of state, and have been adduced as evidence of a fundamental difference in structure between the liquid and gaseous states. Inasmuch as the phenomenon of viscosity provides one of the important distinctions between the ordinary gaseous and ordinary liquid states, it was considered that measurements of this property would be of use in interpreting the transition from liquid to gas at the critical temperature. Several investigations of this nature are described in the literature, notably those of Phillips on carbon dioxide using a capillary flow method (11), Clark on ethyl ether by means of an oscillating cylinder (1, 2), and recently Schröer on carbon dioxide (13) and ethyl ether with a falling ball viscometer. Owing to the limited accuracy of the several experimental techniques, however, these studies must be regarded as being of a preliminary nature.

It was with the purpose of refining and extending these results that the present investigation was undertaken. It was necessary to devise a new experimental technique of measuring viscosities in the critical region to a degree of precision hitherto unattainable. This has been done by means of a modified form of Maxwell's Oscillating Disc (7) whereby viscosities can be measured over a range of temperatures of 0 to 100° C. and up to pressures of 150 atm. to a differential accuracy of 1 : 3000 and an absolute accuracy of 1 : 1000.

Experimental Method

The design of the suspension system of the apparatus is based upon that of Sutherland and Maass (14), and is shown drawn to scale in Fig. 1. The disc is 28.0 mm. in diameter, 0.60 mm. thick, and is made of silver. To the centre of this is brazed a constantan rod 20 mm. long and about 0.3 mm. in

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diameter, soldered to a constantan suspension wire 0.025 mm. in diameter and 100 mm. in length. This is soldered in turn to a rotating head-screw in the perforated brass head. Three small adjusting screws in the brass head are used for mounting the instrument in the containing vessel described below. The silver box is 34 mm. in internal diameter and consists of two silver discs 0.6 mm. thick, clamped together by three small screws and separated by a silver spacing ring 1.80 mm. high. This ring is made in two equal parts, each less than a semicircle to provide two diametrically opposed sight holes in the box. Three long constantan screws 120° apart are threaded through the box, and are firmly held in the head by means of small brass springs. These screws support the box and also serve to adjust its position so that the disc hangs symmetrically about midway between its walls. The oscillations are observed by means of a small glass mirror, 2 mm. square, attached to the constantan rod. Oscillations are induced electro-magnetically through two strips of permalloy fixed to the rod at right angles to the plane of the mirror.

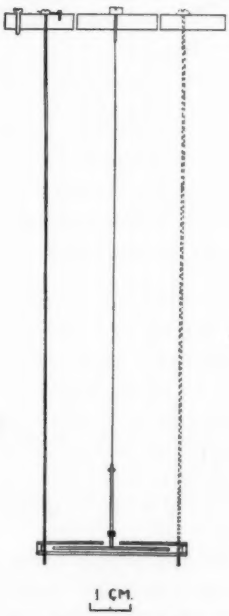

FIG. 1. *The suspension system*

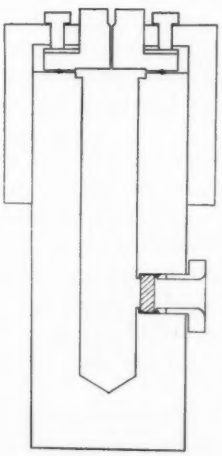

FIG. 2. *The bomb.*

The suspension system rests inside a metal bomb equipped with glass windows, drawn schematically in Fig. 2. The main body was machined from rolled aluminium bronze. It has two diametrically opposed circular windows (only one shown in the diagram) each with a visible diameter of 25 cm. Each window is ½ in. thick and of "Armorplate" glass, and bears on a flat copper gasket in a machined recess of the bomb, and is held in position by a conical copper gasket, a follower, and a hollow nut. The seal is effected by the conical

gasket, and the pressure inside the vessel is utilized to increase the bearing pressure of the window on this gasket. The internal dimensions are such that the suspension system fits snugly, with the supporting screws in the head resting in the recesses in the shoulder at the top of the bomb; the box comes about 1 cm. above the bottom of the visible portion of the window. The head is of aluminium bronze and bears on a recessed copper gasket under the thrust of a steel cap which screws on the main body. Eight $\frac{1}{2}$-in. bolts in the cap bear down on a hardened steel ring and thence upon the head to seal it tight. The head is also provided with a standard $\frac{1}{4}$-in. pressure connection to permit access to the interior of the bomb.

The Bomb Mounting and Assembly

Since the suspension is delicate and susceptible to minute vibration, it was necessary to mount the bomb rigidly to reduce the vibration to a minimum. The main body of the bomb was suspended by means of four $\frac{1}{2}$-in. stainless steel rods from a 6-in. angle wall-bracket imbedded in the solid brick wall of the laboratory. The rods were attached to the bomb through brass angles fixed to four $\frac{1}{2}$-in. bolts extending from the bomb.

The system was assembled by first setting up the bomb on its supports, in a level position and with the top removed. The suspension was placed on its supports and the two sets of screws in the head were used to adjust the relative position of the box and disc, which could be observed through the windows and sight holes in the box. The head was next screwed on and tightened, great care being taken to avoid disturbing the suspension. Finally, the disc was adjusted to the desired position through the hole in the head of the bomb.

The viscometer assembly was set in a large cylindrical copper bath fitted with two rectangular glass windows and fitted with two efficient mechanical stirrers arranged so that no vibration was transmitted to the suspension. The bath was thermostatically controlled manually to within 0.001° C.

To protect the operator from the explosion hazard, the viscometer and thermostat were set up at one end of a substantial wall partition 5 ft. wide by 10 ft. long, and observations were made through a large "Armorplate" glass window at the other end, where all the electrical and thermostatic controls were located.

The disc oscillations were observed on a 100 cm. ground glass scale placed against the window of the partition three metres from the suspension. The scale was corrected for non-uniformity and non-curvature, and was illuminated by a 75 cp. lamp in a standard housing clamped to a rod traversing the width of the partition about two metres from the suspension. A slit was focused on the scale via the mirror, and its position observed through two magnifying glasses sliding on a rod over the complete range of the scale. By moving the lamp housing, the image could be adjusted to the centre of the scale; this compensated for the natural tendency of the suspension to drift.

Two copper coils of several hundred turns and about $1\frac{1}{2}$ in. in diameter were mounted in series on opposite sides of the bomb at right angles to the

windows. By applying 10 volts at 60 cycles, a 100 cm. deflection could be built up in two swings. The damping effect of magnetic hysteresis and induction was eliminated by the use of permalloy, by avoiding the presence of iron in the neighbourhood of the bath and by leaving the exciting coils on open circuit during the free swings of the suspension.

The bomb was illuminated from behind so that the behaviour of the meniscus could be observed through a telescope.

Temperatures were measured by means of a standard Reichanstalt thermometer to within $\pm 0.015°$ C. and a Beckmann thermometer to within $\pm 0.001°$ C. It was necessary to illuminate the thermometers as the room was darkened in the course of a run. A sliding frame carrying a magnifying glass and a small lamp was mounted in front of the thermometers and was moved up and down by means of a string in the hands of the operator. The thermometers were read through a telescope and the magnifying glass, care being taken to avoid parallax.

The Filling System

The system used for measuring and introducing the gases was built on the outside wall of the partition and is shown schematically in Fig. 3. The steel filling bomb, A, is about 80 cc. in volume, capable of withstanding 300 atm., and is wound with a heating coil. The pressure line is $\frac{1}{4}''$–$\frac{1}{16}''$ copper tubing and is brazed to A through a welded T-joint C. E and F are pressure needle valves with standard pressure connections. The low pressure side is of Pyrex and consists of calibrated volumes (H and K), distilling bulbs (L and M), and absolute manometer and McLeod gauge (N and P). The bulb Q is connected to the system through a standard ground glass joint R.

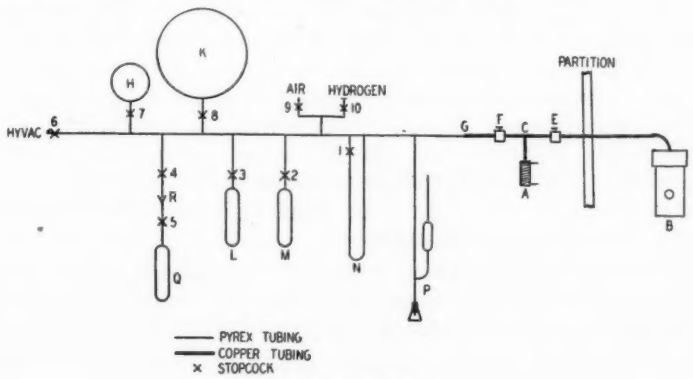

FIG. 3. *The assembly.*

The procedure of introducing samples is as follows. The purified gas is condensed into Q and transferred to a thermostated 50 litre standard volume, measured, and recondensed into Q. It is then condensed into the steel bomb A and the valve F closed, E opened and the bomb slowly heated up

to a temperature of about 80° C. E is then closed and the bomb cooled and F opened and the procedure repeated until the desired quantity is introduced into the viscometer. The residual gas in A and Q is determined by expansion into the standards H and K.

This method of filling, although somewhat primitive, was found to be very satisfactory, and, with experience, quantities within 1% of the desired value could be introduced. The amount of gas can be calculated to within 0.2% by use of gas-law corrections for the respective materials.

Purification of Materials

Air used for calibrating the viscometer was purified by passing it through concentrated potassium hydroxide and sulphuric acid solutions.

Hydrogen, for calibrating the viscometer, and volumes in terms of the standard volume, was prepared from pure granulated zinc and hydrochloric acid in a Kipp generator and was passed through concentrated potassium hydroxide and sulphuric acid solutions, and finally through activated charcoal in liquid air.

Ethylene, on which a number of measurements were made, was purified by fractional distillation in the matter described by Maass and Geddes (4).

Experimental Procedure

Calibration and Manipulation of Viscometer

The viscosity η is calculated from the equation

$$\eta = \frac{\lambda - x}{Ct} \tag{1}$$

where λ is the logarithmic decrement of free oscillations, t is the period (of the order of 30 sec. in the present case), C an instrument constant depending solely upon the geometry of the box-disc system, and x the damping constant of the wire. Equation (1) is in reality an approximation (7) but the instrument dimensions were chosen to make it valid within the experimental accuracy. C is independent of temperature since the suspension system is temperature compensating. The damping factor x varies with temperature, but since it is small compared with λ and the maximum range of temperature was only 15° C., this variation was neglected.

The instrument was calibrated by means of air and hydrogen at 60 cm. pressure and 10.00° C. using Rigden's (12) absolute value for air and Sutherland and Maass's value for hydrogen (14) corrected to Rigden's standard.

A new method with a number of points of superiority over the conventional method devised by Maxwell (7) was used for computing the logarithmic decrement. A maximum deflection of about 100 cm. was built up by means of the exciting coils, and the system allowed to undergo one complete oscillation before any readings were taken. A number of deflections were recorded, and the corresponding time determined on a stopwatch to within 0.2 sec. The amplitudes $\theta_1, \theta_2, \ldots . \theta_m$, were calculated from the corrected scale

readings, m depending on η and so chosen that $\theta_1/\theta_{m+1} \simeq e(2.7)$; in practice m ranged from 3 to 5. The values of $\log \theta_1 - \log \theta_{m+1}$, ... $\log \theta_m - \log \theta_{2m}$ were computed to give m values of $m\lambda$ in decreasing order of accuracy. These values were then multiplied by weights $\overline{w}_1, \overline{w}_2$, ... $\overline{w}_m$ approximately in the ratio $\theta_1 : \theta_2 :$ . . . . $: \theta_m$ and added to give $m(\overline{w}_1 + \overline{w}_2 +$ . . . . . $\overline{w}_m)\lambda$, whence the mean λ was calculated. A sample calculation for the case of air ($m = 5$) at 10.00° C. is given in Table I.

In addition to satisfying the conditions of maximum accuracy this method of weighting the results was found convenient in making rapid calculations, as errors in arithmetic or extraneous disturbances occurring in the course of free oscillations could be detected by comparing the m values of $m\lambda$. When any of the calculated values of $m\lambda$ in a given run differed by more than 1½% from the mean, the run was rejected. The final logarithmic decrements for a given set of conditions were obtained by averaging the results from two to five runs, the actual number of runs depending upon the viscosity and the accuracy required.

TABLE I

SAMPLE DATA FOR METHOD OF CALCULATING λ

N	Scale reading		Corrected scale reading		θ_m	$\log \theta_m$	$\log \theta_m - \log \theta_{m+1}$	$\overline{W}_n$	
	Left	Right	Left	Right					
1	7.77	87.80	8.19	87.51	79.32	1.89938	0.29667	7	2.07669
2	13.07	82.90	13.39	82.66	69.27	1.84054	.29747	6	1.78482
3	17.78	78.61	18.02	78.41	60.39	1.78096	.29737	5	1.48685
4	21.80	74.82	21.98	74.65	52.67	1.72156	.29668	4	1.18672
5	25.33	71.41	25.47	71.33	45.86	1.66143	.29688	3	0.89064
6	28.41	68.59	28.50	68.56	40.06	1.60271		$125\lambda = 7.42572$	
7	31.11	66.12	31.18	66.10	34.92	1.54307		$\lambda = 5.941 \times 10^{-2}$	
8	33.41	63.93	33.47	63.92	30.45	1.48359			
9	35.46	62.11	35.51	62.11	26.60	1.42488			
10	37.22	60.41	37.26	60.41	23.15	1.36455			

Experimental Results

Instrument Constants

In the course of developing the technique, the apparatus was reassembled a number of times and it was necessary to calibrate the suspension each time. The results of a typical calibration are given in Table II. The viscosities of the standards used were 1766.3×10^{-7} and 860.1×10^{-7} poises for air and hydrogen respectively at 10.00° C.

The Discontinuity in Viscosity

Clark (1, 2) has shown that the viscosities of the liquid and gaseous phases of ethyl ether in a stationary bomb in the critical region are dependent upon the thermal history of the system. Although this hysteresis now appears to be less important than originally believed (6, 9), several measurements of this kind were made on ethylene. It was found impractical to make a complete

TABLE II

CALIBRATION DATA FOR VISCOMETER

	λ	t, sec.
Air	5.756×10^{-2}	32.08
Air	5.755×10^{-2}	32.09
Mean	5.755×10^{-2}	32.08_5
Hydrogen	$2.830_7 \times 10^{-2}$	32.07
Hydrogen	$2.830_4 \times 10^{-2}$	32.07
Hydrogen	$2.830_1 \times 10^{-2}$	32.08
Mean	$2.830_4 \times 10^{-2}$	32.07_3

$$C = 10.05_9$$
$$x = 0.05_6 \times 10^{-2}$$

temperature cycle in view of the long time-lags in establishing equilibrium, particularly on the heating portion. Table III gives the results for ethylene in which the bomb was filled, so that the disc was below the meniscus. The system was heated up through the temperature of disappearance of the meniscus (9.50° C.) to 10.00° C. and viscosity determinations were made until no further change could be detected over a period of one-half hour. At this temperature the system came to equilibrium in one and one-half hours. The temperature was then raised to 15° C. and lowered to 10.00° C., and the viscosity was measured and found constant within one-half hour. The results are given in Table III; they indicate a discontinuity of 5%.

TABLE III

INFLUENCE OF THERMAL HISTORY ON VISCOSITY OF ETHYLENE IN THE CRITICAL TEMPERATURE REGION

Temp., ° C.	7.60 up	10.00 up	10.00 down
$\eta \times 10^7$, poises	2907	2137	2037

The Viscosity Isochore

The results for this series are given in Table IV, and are shown plotted to different scales in Figs. 4 and 5. The viscometer, whose volume was found by gas calibration to be 273.5 cc., was filled to an average density of 0.2400 (greater than the critical density), by measuring in 65.63 gm. of purified ethylene. The bath was heated to 26° C. and the temperature lowered through various intervals to 8.50° C. At least an hour was allowed to elapse at each temperature before the viscosity was measured; as found with other properties, approached by cooling down from above the critical temperature it always remained constant. The viscosities are therefore to be considered as true equilibrium values. Particular care was taken between 9.195° C. and 11.000° C., and each viscosity represents the mean of at least three determinations.

The meniscus appeared at the top of the bomb as indicated by effervescence at about 8.3° C. Fig. 4 includes the liquid viscosity at 7.60° C. measured in an independent series (Table III) and shows good agreement with the extrapolated curve.

It will be observed that below the critical temperature, the viscosity of the liquid ethylene decreases rapidly with increase in temperature, and passes through a gradual minimum at 10.0° C., and then rises slowly with increasing temperature as with a normal gas.

TABLE IV

VARIATION OF VISCOSITY OF ETHYLENE (DENSITY, 0.2400) WITH TEMPERATURE

Temp., °C.	26.00	21.00	13.00	11.00$_0$	10.525	10.000	9.475	9.165	8.500
$\eta \times 10^7$, poises	2482	2481	2472	2469	2468	2465	2481	2496	2659

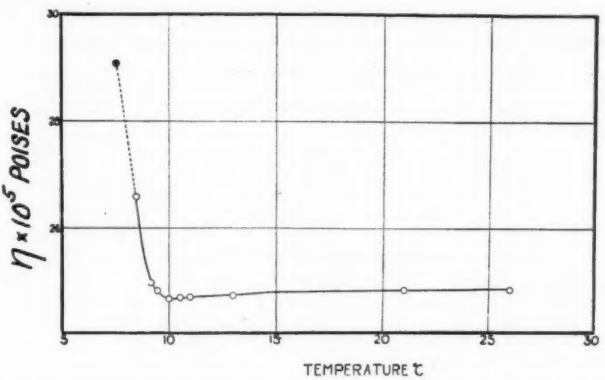

FIG. 4. *Variation of viscosity of ethylene with temperature (density, 0.2400).*

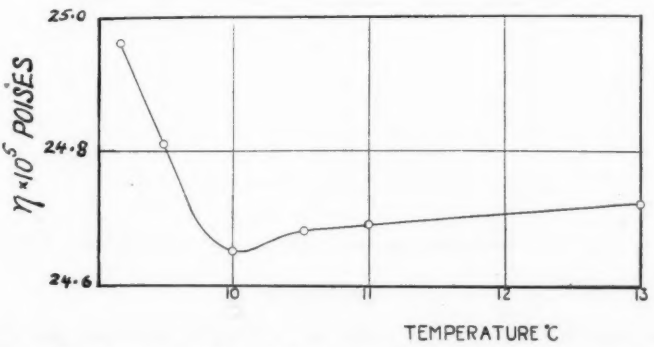

FIG. 5. *Enlarged scale of variation of viscosity of ethylene with temperature just above critical density.*

The Viscosity Isotherm

In this series the bomb was filled to an initial density of 0.3667 by measuring 100.3 gm. of purified ethylene into the system, the bath was thermostatically controlled at 10.00° C. and the viscosity measured. The density was then reduced in a series of steps by successive expansions into the calibrated volumes and the viscosity measured each time, until a final density of 0.18 was reached.

The results are given in Table V and plotted in Fig. 6. The variation of viscosity with density is of the same nature as observed by Phillips (11) and Schröer (13) with carbon dioxide and ethyl ether respectively.

TABLE V

VARIATION OF VISCOSITY OF ETHYLENE WITH DENSITY AT 10.0° C.

Density, gm./cc.	0.367	0.346	0.324	0.303	0.280	0.252	0.222	0.192
$\lambda \times 10^7$, poises	4469	3992	3600	3278	2950	2575	2291	2092

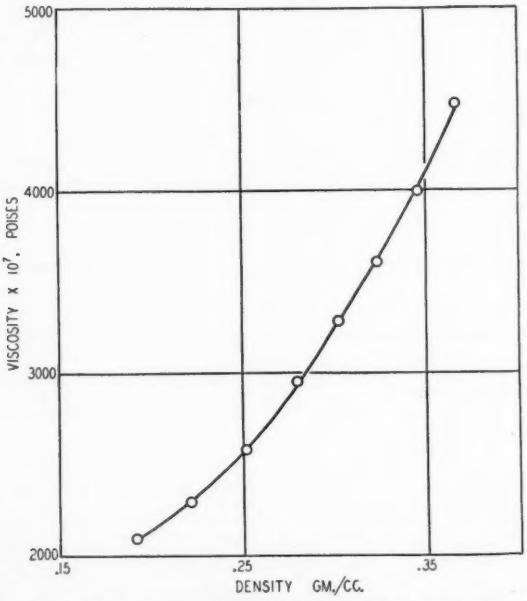

FIG. 6. *Relation between density and viscosity of ethylene at 10° C.*

Conclusion

These experimental results, while not extensive, indicate the advantage of the new technique over existing methods of measuring viscosities at moderately high pressures. It is considered to be premature to discuss their sig-

nificance at the present time. This will be reserved for future communications when more elaborate data on ethylene and other systems will be presented.

It is of considerable interest, however, in the light of previous publications (6, 9) to note the continuous transition in viscosity on the equilibrium isochore illustrated in Fig. 5, and that the minimum viscosity occurs at 10.0° C. The "true" critical temperature of ethylene defined by $\frac{\partial P}{\partial v} = 0$ and $\frac{\partial^2 P}{\partial v^2} = 0$ is 9.90° C. (8), which may tentatively be considered identical with the temperature of minimum viscosity. This suggests that the "true" critical temperature marks the termination of the liquid state, and that a fundamental change in structure accompanies the transition from liquid to gas.

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THE SELENIUM CONTENT OF SASKATCHEWAN WHEAT[1]

By T. Thorvaldson[2] and L. R. Johnson[3]

Abstract

Determinations of the selenium content of 230 composites made up from 2230 individual samples of wheat grown from pure varieties on soils of known type over the whole of the wheat-producing area of the province of Saskatchewan are recorded. The composites were made on the basis of shipping point, wheat variety, and soil type. The maximum amount of selenium found was 1.5 p.p.m., this quantity being present in 10 of the 230 composites, representing 71 (3.2%) of the individual samples. The average found for all the samples analysed was 0.44 p.p.m. It is considered that the averaging that occurs on bulk handling of wheat for export would prevent the selenium content of export cargoes from exceeding materially this average concentration. This represents less than one-tenth of the minimum concentration found to affect the growth of young animals. Analysis of 80 individual samples from the composites containing the highest quantities of selenium showed that no sample contained selenium in concentration as high as the minimum affecting growth. Wheat grown on soil of glacial lacustrine origin predominated among the samples giving the higher selenium content.

Introduction

Chemical analyses of "indicator" plants have shown that selenium is widely distributed in the soils of the central plains of the United States (2) and Canada (1). Soil analyses have confirmed this. Byers and Lakin (1) report one analysis of immature winter rye and four analyses of young wheat plants from the provinces of Saskatchewan and Alberta containing considerable quantities of selenium. Although these authors present no analyses of mature cereal grains, they conclude that there is "a probability of toxic food in certain areas". Before accepting this conclusion, it would seem desirable to obtain further information as to the actual selenium content of mature grains as distinct from the herbage. This paper reports the results of a survey of the selenium content of wheat grown in the province of Saskatchewan.

The Material and Analytical Procedure

There were available more than two thousand samples of wheat of pure varieties, mainly from the 1938 crop (a few samples had been grown in 1936 and 1937), grown on soils of known type over the whole of the wheat producing area of the province*. As the density of the geographical distribution of

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* *This material had been collected by Prof. R. K. Larmour of this university, with the co-operation of the Saskatchewan Wheat Pool and the Saskatchewan Field Husbandry Association, for his studies on the various factors affecting wheat quality. The writers are much indebted to Dr. Larmour for the use of this material.*

these samples corresponds fairly well with the amount of wheat produced in the various districts, this material is well suited for a general survey of the selenium content of the wheat in this province entering into commerce.

As the number of samples was too great for individual analysis, composites were made of all the samples originating at a given shipping point on the basis of the wheat variety and the soil type on which the wheat was grown. The selenium in each of the resulting 230 composites was then determined. Several of the composites showing the highest amounts of selenium were selected and individual determinations were made on all the original samples from which the composite was made up. It might be considered that a composite corresponds to the average of the grain shipped away from a given shipping point, and would be comparable to a sample entering international commerce, while the analyses of individual samples would indicate the maximum selenium content where such mixing as occurs in storing and trans-shipping was absent.

The method of analysis was that described by Robinson, Dudley, Williams, and Byers (7), using the modification of Williams and Lakin (8) for the determination of selenium in organic material. A large number of preliminary determinations were made using a wheat flour, free from selenium, and adding known quantities of the element until the details of the method had been mastered and consistent results were obtained.

As the preparation of a series of standard samples with each batch of determinations is tedious, the use of a photoelectric colorimeter was studied. An Evelyn colorimeter with a 420 filter was employed. Determinations of the rate of development of colour in the presence of 0.010, 0.025, 0.040, and 0.080 mg. of selenium in a 20-gram sample of this control flour (which corresponds to 0.50, 1.25, 2.00, and 4.00 p.p.m.) led to the adoption of the practice of making the readings 24 hr. after the preparation of the test solutions. Curves for the changes in the galvanometer reading with time are given in Fig. 1. The corresponding calibration curve, readings being taken 24 hr.

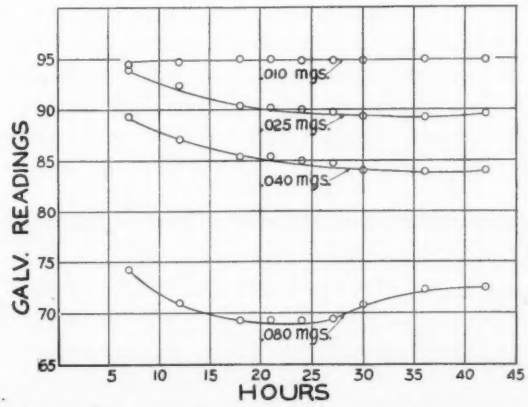

FIG. 1. *The change in colorimeter readings with time for various quantities of selenium.*

after the preparation of the samples, is given in Fig. 2. The relation is almost linear.

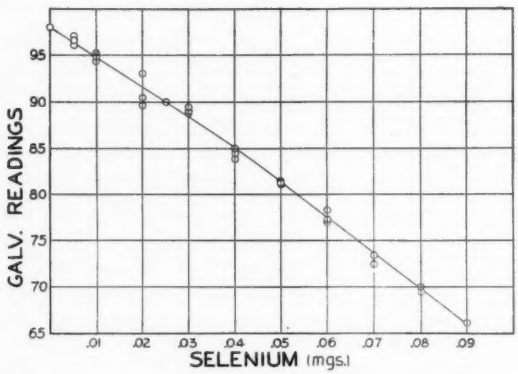

FIG. 2. *Colorimeter readings at 24 hr. for various quantities of selenium.*

A number of comparisons indicated that the use of the photoelectric colorimeter gives at least as reliable results as careful visual comparison with standards. The use of the colorimeter has the advantage of eliminating the personal factor and reducing the time required for the analysis owing to the omission of the preparation of standards with each batch of determinations. The main factors limiting the accuracy of the determination are probably connected with the colloidal nature of the suspension formed, so that the use of the photoelectric colorimeter does not increase the accuracy of the determination. Using 20-gram samples of wheat or flour, it was found that reproducible results agreeing to within 0.25 p.p.m. for the smaller amounts and 0.5 p.p.m. for the larger amounts of selenium were readily obtained.

Nearly all the composite samples of wheat were available in the form of a straight flour milled in an experimental mill. Horn, Nelson, and Jones (4) have shown that the toxic principle in selenium-bearing wheat is present entirely in the protein fraction, and is uniformly distributed in the flour, middlings, and bran milled from the wheat. The determination of selenium was therefore made on the composite straight flour when this was available, but otherwise on the ground whole wheat.

Experimental Results with Wheat Composites

Table I gives the results obtained on the analysis of all the composites, as well as the shipping point of the wheat, the variety, number of individual samples in the composite, and the soil type on which the samples were grown*.

* *The classification of soil types is that used by the Department of Soils at the University of Saskatchewan in their soil surveys (5). The authors are indebted to Prof. J. Mitchell for supplying some of the data from unpublished work. In many cases, samples of wheat grown on several very similar soil types were placed in a single composite.*

TABLE I

SELENIUM CONTENT OF WHEAT COMPOSITES†

Shipping point	Soil type††	Variety of wheat	No. samples in composite	Selenium, p.p.m.
Allan Hills	WL; WCL-L	Thatcher	13	0.5
Alticane C1	OCL	Thatcher	2	0.25
Alticane C2	OL	Thatcher	14	0
Ambassador	WL-LL	Thatcher	15	0.5
Antler	YL and OL	Thatcher	18	0
Arcola	OL	Thatcher	4	0.25
Arran	PCL	Marquis	12	0.5
Assiniboia	HrCL-L	Marquis	12	1.0
Battleford*C1 (1936)	MeFSL and OL	Marquis	4	0.25
Battleford*C2 (1936)	WL and OL; WvL and OL	Marquis	11	0.5
Bateman C1	HrL; FxSiL; HrSiL-L	Marquis	10	<0.25
Bateman C2	FxSiL; ScC; HrCL	Marquis	9	1.0
Bear Creek	WvL	Thatcher	23	0.5
Bengough	HrL; HrCL	Thatcher	9	0.5
Bethune	WLL-L	Thatcher	23	0.5
Bladworth C1	WL	Marquis	13	0.5
Bladworth C2	WL and EcL	Marquis	4	0.25
Bounty	EcL; ESiCL	Marquis	10	0
Briarlea C1	SbFSL	Marquis	6	0.5
Briarlea C2	SbL-FSL	Marquis	7	0
Brooksby	TiSiCL	Thatcher	15	<0.25
Bruno	WL-LL; ESiL	Reward	10	0.25
Buffalo Gap	HrL	Thatcher	8	0.5
Buchanan C1*(1937)	GSL	Thatcher	14	0.25
Cadillac C1	HrL; HrL-LL	Marquis	6	0.5
Cadillac C2	HrL; HtFSL	Thatcher	4	0.25
Cadillac C3	HrL; HrL-LL	Marquis	3	0
Cadillac C4	HrL; HrL-LL	Red Bobs	9	0.25
Calder	YL	Marquis	4	0.25
Cana	OL and YL	Thatcher	8	0.5
Cando	WL	Marquis	15	0.5
Caron C1	WL	Marquis	7	0.5
Caron C2	RHvC-CL	Marquis	6	0.5
Ceylon C1	HrCL and WCL	Thatcher	4	1.0
Ceylon C2	WL and EcCL	Thatcher	3	1.5
Churchbridge	YL	Thatcher	13	0.25
Clair	YL-LL	Thatcher	8	0.5
Colonsay	EC and ESiCL	Thatcher	11	0.25
Corning	OL and WL	Thatcher	16	0.25
Courval	HrL; and HrCL	Marquis	7	0.5
Craik	WL	Thatcher	10	1.0
Cudworth C1	OLL-SbFSL; SbLL	Thatcher	10	0.5
Cudworth C2	OSiL and BSiL	Thatcher	10	1.0
Cupar C1	OL	Marquis	9	0.25

† All samples were grown in 1938 except where designated by the year in brackets, i.e., (1937). All analyses were made on the straight wheat flour except those marked with an asterisk, the analysis in these cases being made on the ground whole wheat. Where two or more composites were made of wheat from a single shipping point they are marked C1, C2, etc.

†† Key to symbols for soil type:—

Soil series: A—*Asquith;* B—*Blaine Lake;* Cy—*Cypress;* E—*Elstow;* Ec—*Echo;* Fx—*Fox Valley;* Hr—*Haverhill;* Ht—*Hatton;* M—*Melfort;* Me—*Meota;* O—*Oxbow;* P—*Pelly;* R—*Regina;* Sc—*Sceptre;* Sb—*Shellbrook;* Ti—*Tisdale;* W—*Weyburn;* Wv—*Waitville;* Y—*Yorkton.* "Alk" *indicates presence of "white alkali".*

Soil class: C—*clay;* HvC—*heavy clay;* CL—*clay loam;* SiCL—*silty clay loam;* L—*loam;* LL—*light loam;* FSL—*fine sandy loam;* VFSL—*very fine sandy loam;* GSL—*gravelly sandy loam.*

TABLE I—*Continued*

SELENIUM CONTENT OF WHEAT COMPOSITES†—*Continued*

Shipping point	Soil type††	Variety of wheat	No. samples in composite	Selenium, p.p.m.
Cupar C2	WL-LL and OL-LL; OCL-L and WCL-L	Marquis	4	0.25
Deer Creek C1	GSL	Reward	8	0
Deer Creek C2	WvL	Reward	7	1.0
Delisle	ALL and AFSL	Thatcher	6	0
Demaine C1	HrL	Thatcher	5	1.0
Demaine C2	HrCL	Thatcher	5	0.5
Dilke	WL-LL	Marquis	25	0.5
Drake C1	WFSL and AFSL; WLL	Thatcher	5	0.25
Drake C2	WL and OL	Thatcher	8	0.5
Earl Grey	OL-LL and WL-LL	Marquis	16	<0.25
Edenwold	OL and WL	Thatcher	14	0.5
Elbow	WL	Reliance	15	0.5
Eldersley	TiSiCL	Thatcher	12	<0.25
Elrose	RHvC	Marquis	11	1.5
Elstow	EC and ESiCL; ESiL	Marquis	5	<0.25
Ernfold	HrL; HrCL-L	Thatcher	18	0.5
Evesham C1	WL	Marquis	11	0.25
Evesham C2	WCL-L; ECL and WL	Marquis	19	0.5
Eston	ScHvC	Marquis	15	1.0
Ferland	HrLL and HrFSL	Marquis	6	0.5
Fillmore	WL; WLL-L	Thatcher	17	<0.25
Foam Lake	OLL-L and YLL-L	Thatcher	16	0.5
Forgan C1	RHvC; RC and ESiCL	Marquis	10	1.5
Forgan C2	RHvC; RC and ESiCL	Marquis	4	1.0
Francis	WL	Thatcher	8	1.0
Freemont	OL	Marquis	15	0.25
Frenchman's Butte	SbGSL	Thatcher	11	0.5
Girvin	WL; WL-LL	Reward	9	0.5
Glen Bain C1	HrL	Marquis	11	1.0
Glen Bain C2	ScC and HrCL	Marquis	4	1.0
Glen Ewen	OLL and WLL; OSiL	Thatcher	15	0.25
Gorlitz	MeLL-FSL	Thatcher	7	<0.25
Govan	WL; WLL-L	Thatcher	20	0.25
Gravelbourg C1	HrL	Thatcher	10	0.25
Gravelbourg C2	HrCL-SiL	Thatcher	7	0.5
Grenfell	OL	Renown	11	0.5
Hague C1	OL	Thatcher	8	<0.25
Hague C2	MeLL; MeFSL	Thatcher	7	0
Hanley	WL	Marquis	11	<0.25
Hawarden C1	WL	Thatcher	23	0.5
Hawarden C2	WL	Thatcher	5	0.25
Hearne	RHvC and ESiCL	Thatcher	9	1.5
Hendon	YL-LL	Renown	16	0

† *All samples were grown in 1938 except where designated by the year in brackets, i.e., (1937). All analyses were made on the straight wheat flour except those marked with an asterisk, the analysis in these cases being made on the ground whole wheat. Where two or more composites were made of wheat from a single shipping point they are marked C1, C2, etc.*

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Soil class: C—*clay;* HvC—*heavy clay;* CL—*clay loam;* SiCL—*silty clay loam;* L—*loam;* LL—*light loam;* FSL—*fine sandy loam;* VFSL—*very fine sandy loam;* GSL—*gravelly sandy loam.*

TABLE I—*Continued*

SELENIUM CONTENT OF WHEAT COMPOSITES†—*Continued*

Shipping point	Soil type††	Variety of wheat	No. samples in composite	Selenium, p.p.m.
Hepburn C1	OL; OLL	Marquis	17	0.5
Hepburn C2	BCL	Marquis	8	0.5
Herschel C1	HrL; HrCL-L	Thatcher	5	0.5
Herschel C2	HrL and WL; Alk.	Thatcher	2	1.0
Herschel C3	HrL; HrL-CL	Thatcher	6	1.0
Herschel C4	HrL and WL; Alk.	Thatcher	3	1.5
Hubbard	OL and YL-LL	Thatcher	13	<0.25
Holbein	SbL-FSL; SbFSL	Apex	17	0.25
Indian Head	Ind. Head C; Ind. Head CL	Thatcher	14	0.5
Insinger C1	OLL-L and YLL-L	Thatcher	9	0
Insinger C2	YL and WvL; WvL	Thatcher	12	0.5
Instow	HrL and CyL	Marquis	20	0.5
Jansen C1	WL and OL	Marquis	5	0.25
Jansen C2	YL-LL; WL-LL (Alk)	Marquis	4	0.5
Kelvington	PCL-L	Thatcher	8	0.5
Kennedy C1	OL; OL and YL	Thatcher	8	0.25
Kennedy C2	OL and WL	Thatcher	9	0.5
Kinistino	MSiCL and WvL	Renown	11	<0.25
Kinley C1	AFSL	Marquis	6	0.25
Kinley C2	ESiL	Marquis	6	0.5
Kuroki	YL-LL	Thatcher	6	<0.25
Lancer* (1936)	ScHvC	Marquis	15	0.5
Landis C1	WL	Marquis	6	0.5
Landis C2	WL	Marquis	16	1.0
Landis C3	ESiL	Marquis	7	0.5
Landis C4	ESiL	Marquis	8	0.5
Langham	ALL-FSL	Marquis	18	0
Laporte C1	HrL and EcL; EcCL-L	Marquis	10	0.25
Laporte C2	ScHvC	Marquis	5	0
Lawson	EcCL-L	Thatcher	15	0.5
Leader and Sceptre	ScC; ScHvC	Marquis	18	1.0
Leask C1 and Parkside	SbFSL	Thatcher	17	<0.25
Leask C2	BSiL-L; MeLL	Thatcher	9	0.25
Lepine C1	BSiL and WvL	Marquis	7	0.25
Lepine C2	BSiL and WvL	Marquis	3	0.5
Le Roy	OL and YL; YL-LL	Thatcher	13	0.25
Livelong C1	WvL	Thatcher	3	0.5
Livelong C2	OL	Thatcher	3	<0.25
Lipton	OL and YL-LL; OL and YLL	Thatcher	9	<0.25
Lone Rock	OL	Red Bobs	15	0
Luseland C1*(1936)	ECL and WL	Marquis	10	<0.25
Macoun	WL	Marquis	11	0
Mankota	HrL	Marquis	21	1.0
Marchwell C1	YL	Renown	7	0.5

† *All samples were grown in 1938 except where designated by the year in brackets, i.e., (1937). All analyses were made on the straight wheat flour except those marked with an asterisk, the analysis in these cases being made on the ground whole wheat. Where two or more composites were made of wheat from a single shipping point they are marked C1, C2, etc.*

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Soil class: C—*clay:* HvC—*heavy clay;* CL—*clay loam;* SiCL—*silty clay loam;* L—*loam;* LL—*light loam;* FSL—*fine sandy loam;* VFSL—*very fine sandy loam;* GSL—*gravelly sandy loam.*

TABLE I—*Continued*

SELENIUM CONTENT OF WHEAT COMPOSITES†—*Continued*

Shipping point	Soil type††	Variety of wheat	No. samples in composite	Selenium, p.p.m.
Marchwell C2	OLL	Renown	5	0.5
Margo	YL-LL	Apex	12	0.5
Mawer C1	HrL; HrCL-L	Thatcher	11	0.25
Mawer C2	WL-LL	Thatcher	8	0.5
Mazenod C1	ScHvC	Marquis	6	0.5
Mazenod C2	HrCL-L	Marquis	5	<0.25
Maxstone	HrCL-L	Marquis	8	0.25
McTaggart C1	RHvC	Thatcher	2	1.0
McTaggart C2	RCL	Thatcher	2	1.5
McTaggart C3	EcL and WL	Marquis	2	<0.25
Meadow Lake	ML	Reward	8	0.5
Melfort	MSiCL and WvL	Thatcher	8	0.25
Mendham	FxSiCL; HrL and FxL	Marquis	17	0.5
Milestone C1*(1936)	RHvC	Reward	4	0.5
Milestone C1*(1936)	RHvC	Marquis	10	0.25
Montmartre	WL; OL	Thatcher	11	0.25
Moose Jaw C1*(1937)	RHvC; WL	Marquis	10	0.25
Neidpath and Waldeck	HrL; HrCL-L	Marquis	12	1.0
Neudorf	OL and YL-LL; OL	Thatcher	6	<0.25
Norquay C1*(1937)	TiSCL and PC	Not known	14	0.25
Osler C1	OL	Marquis	4	0.5
Osler C2	AFSL	Marquis	2	<0.25
Oxbow C1	MFSL and AFSL	Thatcher	3	0.25
OxBow C2	WL and OL	Thatcher	3	0.25
Pangman	WL and HrL; EcGL and HrCL	Thatcher	18	<0.25
Paynton	MeLL; MeFSL	Marquis	5	0.5
Pennant C1	HtFSL	Marquis	9	0.5
Pennant C2	HrL-CL; FxSiCL	Marquis	6	0.5
Pennant C3	ScHvC	Marquis	3	1.0
Pennant C4	Alk	Marquis	3	1.5
Percival	OL and WvL	Thatcher	18	0.25
Peterson	WL; WL and ESiL	Marquis	12	<0.25
Porcupine	TiL	Thatcher	10	<0.25
Prud'Homme	WSiL and ESiL; WL	Thatcher	12	0.5
Punnichy C1*(1937)	OL and WL; ALL-FSL	Marquis	8	0.25
Readlyn C1	ScHvC	Thatcher	17	1.5
Readlyn C2	HrCL and HrCL-L	Thatcher	7	1.0
Regina C1*(1937)	RHvC	Thatcher	8	0.5
Riverhurst	HtLL-FSL; HrLL	Marquis	11	0.5
Rocanville	OL-LL	Thatcher	10	0.5
Robsart	EcCL and HrL	Marquis	7	0.5
Runnymede C1	WvL	Thatcher	5	0.25
Runnymede C2	PL	Thatcher	7	0.5

† *All samples were grown in 1938 except where designated by the year in brackets, i.e., (1937). All analyses were made on the straight wheat flour except those marked with an asterisk, the analysis in these cases being made on the ground whole wheat. Where two or more composites were made of wheat from a single shipping point they are marked C1, C2, etc.*

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TABLE I—*Continued*

SELENIUM CONTENT OF WHEAT COMPOSITES†—*Continued*

Shipping point	Soil type††	Variety of wheat	No. samples in composite	Selenium, p.p.m.
Ruthilda C1	WL	Marquis	6	1.0
Ruthilda C2	RHvC	Marquis	2	1.0
Ruthilda C3	WFSL	Marquis	2	1.0
Ryerson and Fairlight	OL and YL	Thatcher	29	0.5
Scottsburgh C1	HrLL; HrL; HrCL-L	Thatcher	8	0.5
Scottsburgh C2	Alk	Thatcher	2	<0.25
Scout Lake C1	HrL and CyL	Thatcher	14	0.5
Scout Lake C2	HrCL-L	Thatcher	13	1.0
Sintaluta	OL	Thatcher	13	0
Shamrock	HrL	Marquis	14	0.5
Sheho	WvL	Thatcher	13	0.5
Sonningdale C1	ALL-FSL	Marquis	9	<0.25
Sonningdale C2	BSiL	Marquis	4	0.25
Sovereign C1*(1937)	ESiCL; RHvC	Marquis	8	0.5
Spalding	YL	Thatcher	30	0.25
Spy Hill	YL; GSL	Thatcher	8	0.25
Stewart Valley C1* (1936)	HrCL	Marquis	4	0.25
Stewart Valley C2* (1936)	HrL	Marquis	2	0.5
Stewart Valley C3* (1936)	ScHvC	Marquis	5	1.0
Stockholm	OL and YL	Thatcher	17	<0.25
Strongfield	WL	Marquis	22	<0.25
Struan C1	ALL-FSL	Marquis	14	0.25
Struan C2	OL and WvL-FSL	Marquis	4	1.0
Struan C3	EL; ESiL; BSiL	Marquis	3	0.5
Swanson C1	ALL-FSL	Marquis	14	0.5
Swanson C2	AFSL	Marquis	9	0.5
Sylvania C1	WvLL-FSL	Thatcher	9	0.25
Sylvania C2	TiSiCL and WvL	Thatcher	8	0.25
Sylvania C3	TiSiCL	Thatcher	6	1.0
Tako C1	WL-LL;ALL-FSL	Marquis	11	<0.25
Tako C2	WL-LL; ALL-FSL-L	Marquis	5	0.25
Talmage	WL and EcL; EcCL-WL	Thatcher	16	0.5
Tantallon C1	YLL-L; OL and YL	Thatcher	7	0.5
Tantallon C2	OLL	Thatcher	7	0.5
Tugaske	WL; WL-LL	Thatcher	19	0.25
Tuxford and Marquis	RHvC; RHvC-CL	Marquis	20	0.5
Vanguard	ScC and HrCL	Marquis	9	1.0
Viscount C1	WL	Marquis	3	0.25
Viscount C2	EC-SiCL	Marquis	3	1.0
Viscount C3	WCL and ECL	Marquis	2	0.25
Viscount C4	WCL and ECL	Thatcher	2	0.5

† All samples were grown in 1938 except where designated by the year in brackets, i.e., (1937). All analyses were made on the straight wheat flour except those marked with an asterisk, the analysis in these cases being made on the ground whole wheat. Where two or more composites were made of wheat from a single shipping point they are marked C1, C2, etc.

†† Key to symbols for soil type:—

Soil series: A—*Asquith;* B—*Blaine Lake;* Cy—*Cypress;* E—*Elstow;* Ec—*Echo;* Fx—*Fox Valley;* Hr—*Haverhill;* Ht—*Hatton;* M—*Melfort;* Me—*Meota;* O—*Oxbow;* P—*Pelly;* R—*Regina;* Sc—*Sceptre;* Sb—*Shellbrook;* Ti—*Tisdale;* W—*Weyburn;* Wv—*Waitville;* Y—*Yorkton.* "Alk" *indicates presence of "white alkali".*

Soil class: C—*clay;* HvC—*heavy clay;* CL—*clay loam;* SiCL—*silty clay loam;* L—*loam;* LL—*light loam;* FSL—*fine sandy loam;* VFSL—*very fine sandy loam;* GSL—*gravelly sandy loam.*

TABLE I.—*Concluded*

SELENIUM CONTENT OF COMPOSITES†—*Concluded*

Shipping point	Soil type††	Variety of wheat	No. samples in composite	Selenium, p.p.m.
Viscount C5	WCL and ECL	Reward	2	1.0
Watson	YL-LL	Thatcher	8	0.5
Wawota	OL-LL; OL and YL	Thatcher	19	0.25
Webb C1	HrL	Marquis	12	0.5
Webb C2	CyL	Marquis	9	0.5
Webb C3	HrFSL-VFSL	Marquis	11	1.0
Weirdale C1	TiCL	Thatcher	5	0
Weirdale C2	TiCL	Thatcher	4	0.25
Welwyn	OL and YL	Thatcher	17	<0.25
White Bear C1	ScHvC	Marquis	7	1.5
White Bear C2	HrCL-L and ScHvC	Marquis	6	1.5
Whitewood	OL and YL	Thatcher	15	0.5
Wolseley	OL	Thatcher	15	0
Woodrow	HrCL	?	3	1.0
Zelma	ESiCL; ECL	Marquis	20	0.5

† *All samples were grown in 1938 except where designated by the year in brackets, i.e., (1937). All analyses were made on the straight wheat flour except those marked with an asterisk, the analysis in these cases being made on the ground whole wheat. Where two or more composites were made of wheat from a single shipping point they are marked C1, C2, etc.*

†† *Key to symbols for soil type:—*

Soil series: A—*Asquith;* B—*Blaine Lake;* Cy—*Cypress;* E—*Elstow;* Ec—*Echo;* Fx—*Fox Valley;* Hr—*Haverhill;* Ht—*Hatton;* M—*Melfort;* Me—*Meota;* O—*Oxbow;* P—*Pelly;* R—*Regina;* Sc—*Sceptre;* Sb—*Shellbrook;* Ti—*Tisdale;* W—*Weyburn;* Wv—*Waitville;* Y—*Yorkton.* "Alk" *indicates presence of "white alkali".*

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Fig. 3 shows the approximate locations at which wheat samples were grown on an outline map of the province of Saskatchewan showing the general soil zones and the selenium content of each composite represented by symbol. In cases where more than one symbol appears at a given location on the map more than one composite, on the basis of soil type and wheat variety, had been prepared from samples of wheat received from a single shipping point.

The data of Table I show that the highest amount of selenium found in any wheat composite was 1.5 p.p.m. The maximum amount found is therefore well below the lower limit suggested in the literature as toxic to animals. Franke and Painter (3, p. 22) in the report of a five year study published in 1938, state that "any diet containing over 5 p.p.m. selenium will retard the growth of young animals". They found that diets containing more than 9 p.p.m. of selenium cause death in young animals, while older animals are much more resistant to selenium poisoning. There was a pronounced restriction of food consumption in diets containing over 10 p.p.m. of selenium. These conclusions were based on studies on the rat. Munsell, De Vaney, and Kennedy (6), from their study with rats, report that on a diet containing 6 p.p.m.

of selenium, weight was considerably below normal and the number of young born and the percentage reared were less than normal. A diet containing 3.0 p.p.m. had a slight effect on reproduction although growth was normal. When the diet contained 1.5 p.p.m. or less, there was no detectable effect on growth or reproduction.

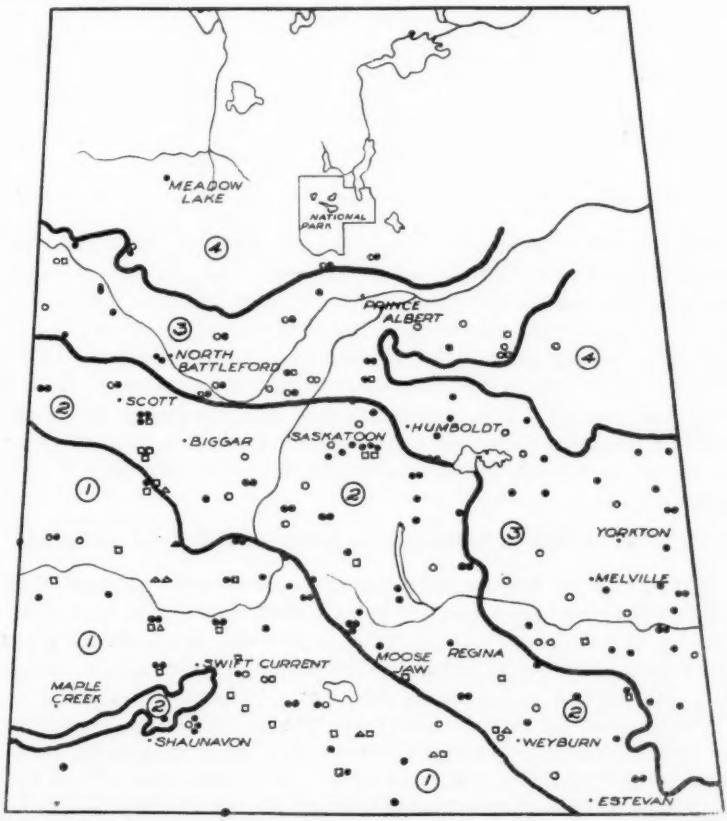

FIG. 3. *Map of Saskatchewan showing locations at which wheat samples were grown and the selenium content of the composites. Symbols for selenium content:* ○ *= less than 0.25 p.p.m.;* ● *= 0.25 to 0.5 p.p.m.;* □ *= 1.0 p.p.m.;* △ *= 1.5 p.p.m. General soil zones: 1. Brown soil; short prairie grass; 2. Dark brown soil; intermediate prairie region; 3. Black soil; tall grass, Park region; 4. Grey soil; wooded region.*

Table II gives the distribution of the composites and of the individual samples represented by them according to selenium content. Of the 230 composites only 10 (representing 3.2% of the individual samples of wheat) contained more than 1 p.p.m., while 99 composites (43% of the composites representing 44% of the samples) contained 0.25 p.p.m. or less of selenium. The average value for all the composites was 0.46 and for all the individual samples 0.44 p.p.m.

TABLE II

SUMMARY OF SELENIUM CONTENT OF COMPOSITES

Selenium content, p.p.m.	No. of composites	Per cent composites	No. of individual samples	Per cent total
1.5	10	4.3	71	3.2
1.0	33	14.3	252	11.3
0.5	88	38.3	926	41.5
0.25	53	23.0	498	22.2
<0.25	29	12.6	303	13.6
Absent	17	7.4	180	8.1
Total	230		2230	

Considering the large number of samples and their broad distribution over the wheat growing area, the results of the survey indicate that it is very unlikely that a cargo of wheat grown in the province of Saskatchewan, which normally would be a composite of grain from a number of shipping points and subject to bulk handling in local and terminal elevators and cargo boats, would be found to contain appreciably more than the average value, namely, 0.44 parts selenium per million. This represents less than one-tenth of the lowest estimate of the quantity of selenium considered to have any effect on the growth of young animals even if the whole diet were composed of wheat of this concentration.

Determination of Selenium in Individual Wheat Samples

In the case of grain fed to stock on the farm or ground into flour at a small local mill, the averaging that occurs in bulk handling of grain for export may be to a large extent absent. It was therefore desirable to make determinations of the selenium content of the individual wheat samples of the composites showing the highest quantity of selenium. Unfortunately, some of the individual samples had been discarded after the composites had been prepared. Of the 71 individual wheat samples contained in the 10 composites showing a selenium content of 1.5 p.p.m., only 41 were still available. These were all analysed. In addition, the selenium content of 39 individual samples from the composites showing 1.0 p.p.m. was determined. The results are shown in Table III.

The maximum quantities of selenium found in individual samples were 4.0 p.p.m. for the composites containing 1.5 p.p.m., and 3.0 p.p.m. for the composites containing 1.0 p.p.m., in each case only one sample containing these quantities in the 40 samples analysed. It is apparent that in no case did the selenium content of single samples reach the minimum suggested as harmful to the growth of young animals even if the whole diet were composed of the wheat. In only one sample did the quantity of selenium exceed the threshold value suggested as affecting slightly the reproduction of young animals, and in no case if the diet contained not over 75% of the wheat. It

would, however, be desirable to investigate more thoroughly the incidence of selenium in individual grain samples as well as in straw and forage in the districts showing the highest selenium content in wheat.

TABLE III

DETERMINATION OF SELENIUM IN INDIVIDUAL WHEAT SAMPLES

Selenium, found, p.p.m.	No. of samples from composites containing	
	1.5 p.p.m.	1.0 p.p.m.
4.0	1	0
3.0	3	1
2.5	3	1
2.0	11	4
1.5	9	9
1.0	9	8
0.5	5	14
0.25	0	2
Total	41	39

Relation of the Occurrence of Selenium to the Soil Type

Byers and his co-workers have found "a close correlation between the geological strata from which the soils have developed and the quantity of selenium found in the soils" (1). As the soils of the wheat-growing area of Saskatchewan are generally of glacial origin, it is difficult to relate them to preglacial deposits. They have, however, been surveyed and classified (5) into zones, each zone being in general characterized by its type of climate and its natural vegetation. Further subdivision of the soil zone into series is based on profile characteristics. The soils of any one series are derived from similar parent geological materials. Each series is further subdivided into soil types based on the surface texture of the soil. The soil type is the unit used in the classification, and the name of any type is derived by combining the series name with the surface texture, as "Regina heavy clay" or "Haverhill clay loam."

A study was made of the data of Table I as to the percentage frequency of the occurrence of selenium in quantities of 0, <0.25, 0.25, 0.50, 1.0, 1.5 p.p.m. in wheat grown on soils texturally classified as heavy clay, clay, clay loam, loam and light loam. The frequency curves showed a shift towards the higher concentrations of selenium in the order: light loam, loam, clay loam, clay, heavy clay. The maxima of the frequency curves appeared at 0.5% of selenium for the wheat grown on light loam, loam, and clay loam, but at higher concentrations for those grown on clay and heavy clay. This indicates that wheat grown on soils formed from glacial lacustrine deposits tends to higher selenium content.

Percentage frequency curves were also plotted for the frequency of the occurrence of the same amounts of selenium in wheat grown on the following soil series: Yorkton, Oxbow, Weyburn, and Haverhill. It was found that the curves were more irregular but for the higher concentrations of selenium there was an increase in the frequency in the order: Yorkton, Oxbow, Weyburn, Haverhill. Prof. John Mitchell has suggested to the authors that the marked difference in the frequency curves for the wheat grown on the Yorkton and Haverhill soil series may possibly be due to the greater influence of the material of the underlying preglacial deposits in the latter series on account of the shallower covering of glacial drift.

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